Module 1
Quantitative Reliability Analysis

Chanan Singh
Texas A&M University

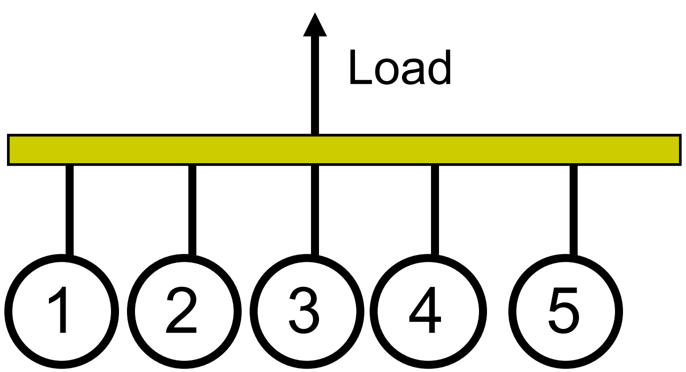
Introduction to Quantitative Reliability

- Reliability relates to the ability of a system to perform its intended function
- Qualitative vs. quantitative concept of reliability.
- When quantitatively defined, reliability becomes a parameter that can be traded off with other parameters like cost
- Necessity of quantitative reliability : decision making
  - Ever increasing complexity of system design and operation
  - Evaluation of alternate design proposals
  - Cost competitiveness and cost-benefit trade off

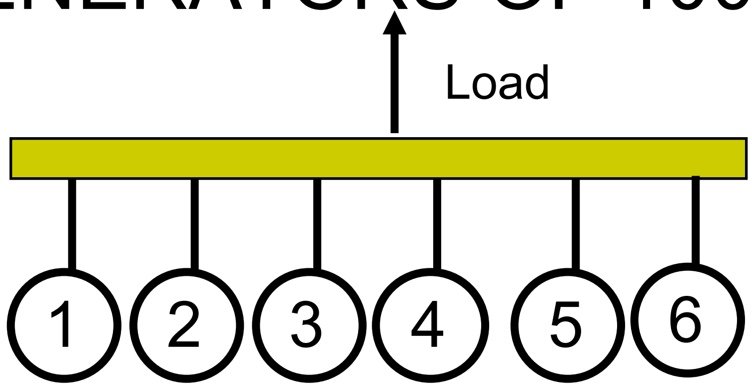
Discussion Questions

- WE HAVE A LOAD OF 500 MW
- WHICH OF THE FOLLOWING ALTERNATIVES IS THE BEST CONSIDERING COST AND RELIABILITY?

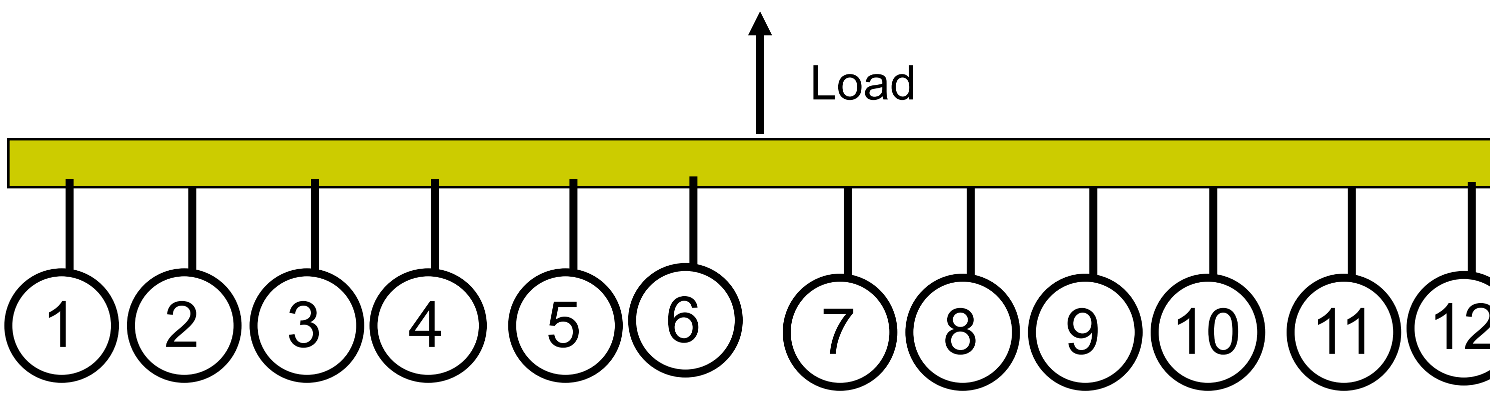
- 5 GENERATORS 100 MW EACH



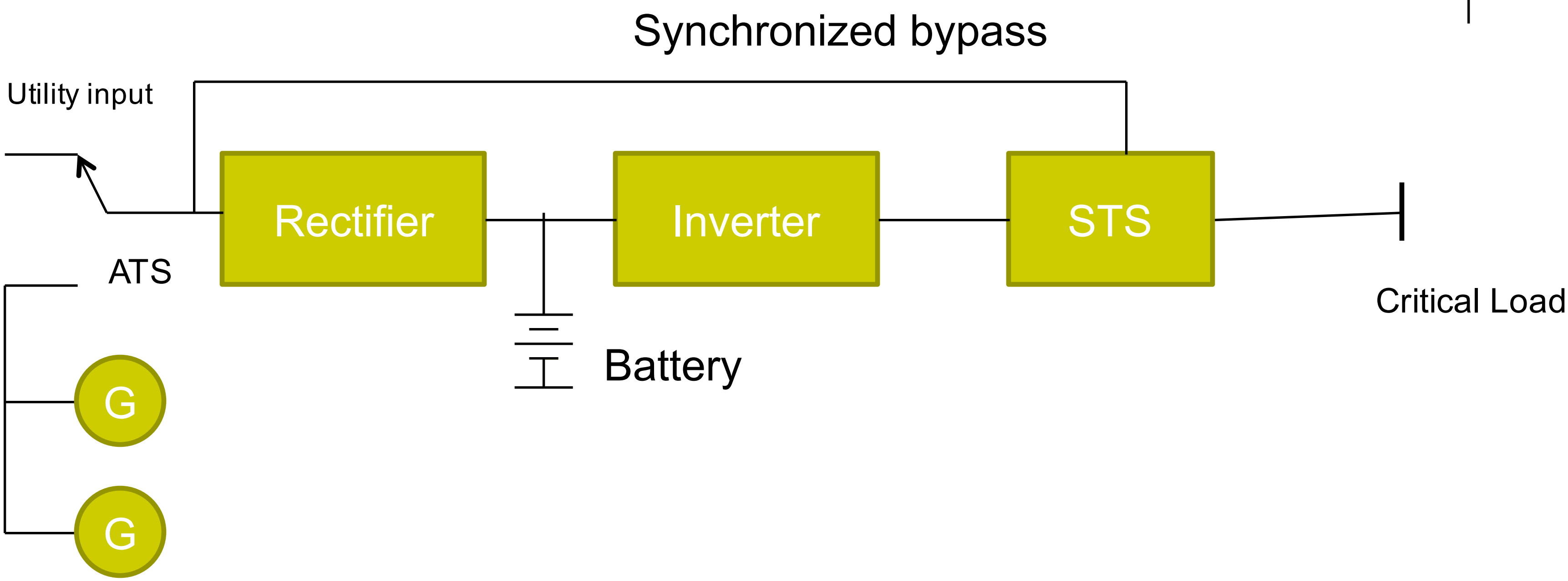
- 6 GENERATORS OF 100 MW EACH



-12 GENERATORS OF 50 MW EACH



Standby Power



Measures of Reliability Quantification

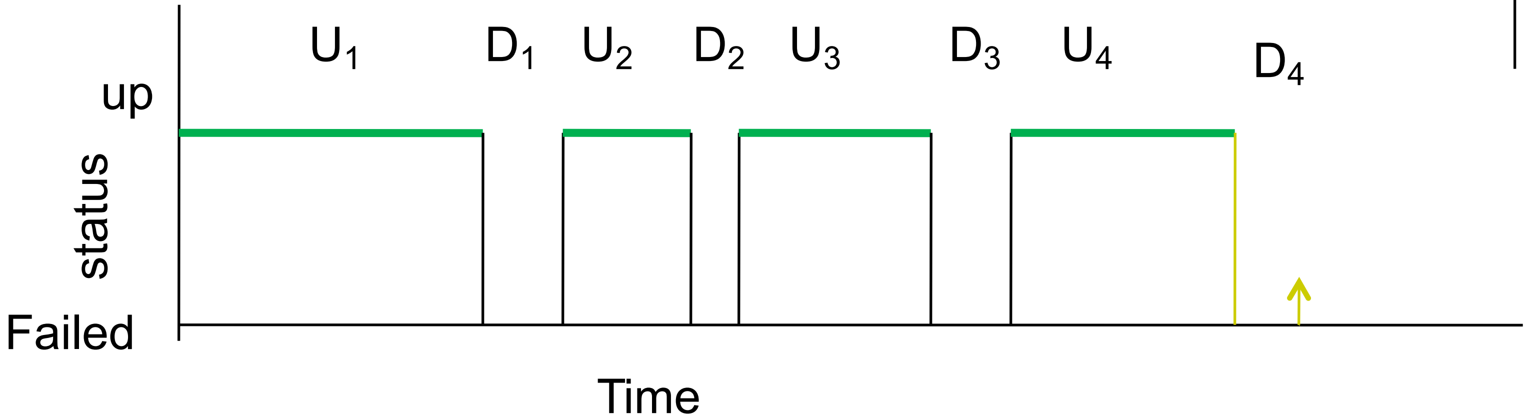
- Basic indexes
  - Probability of failure

    Long run fraction of time system is failed
  - Frequency of failure

    Expected or average number of failures per unit time
  - Mean duration of failure

    Mean duration of a single failure
- Other indexes can be generally obtained as a function of the above.

Example of Indices



Up Times	Down Times	Cycle Time
$U_1 = 100$ h	$D_1 = 10$ h	110
$U_2 = 50$ h	$D_2 = 5$ h	55
$U_3 = 80$ h	$D_3 = 6$ h	86
$U_4 = 90$ h	$D_4 = 4$ h	94
Total= 320 h	25h	320+25=345

Estimate of probability of failure = 25/(320+25) = .072

Estimate of frequency of failure (FF) = 4/345 = .011594 f/h=101.56 f/y

Estimate of mean down time(MDT) = 25/4 = 6.25 h

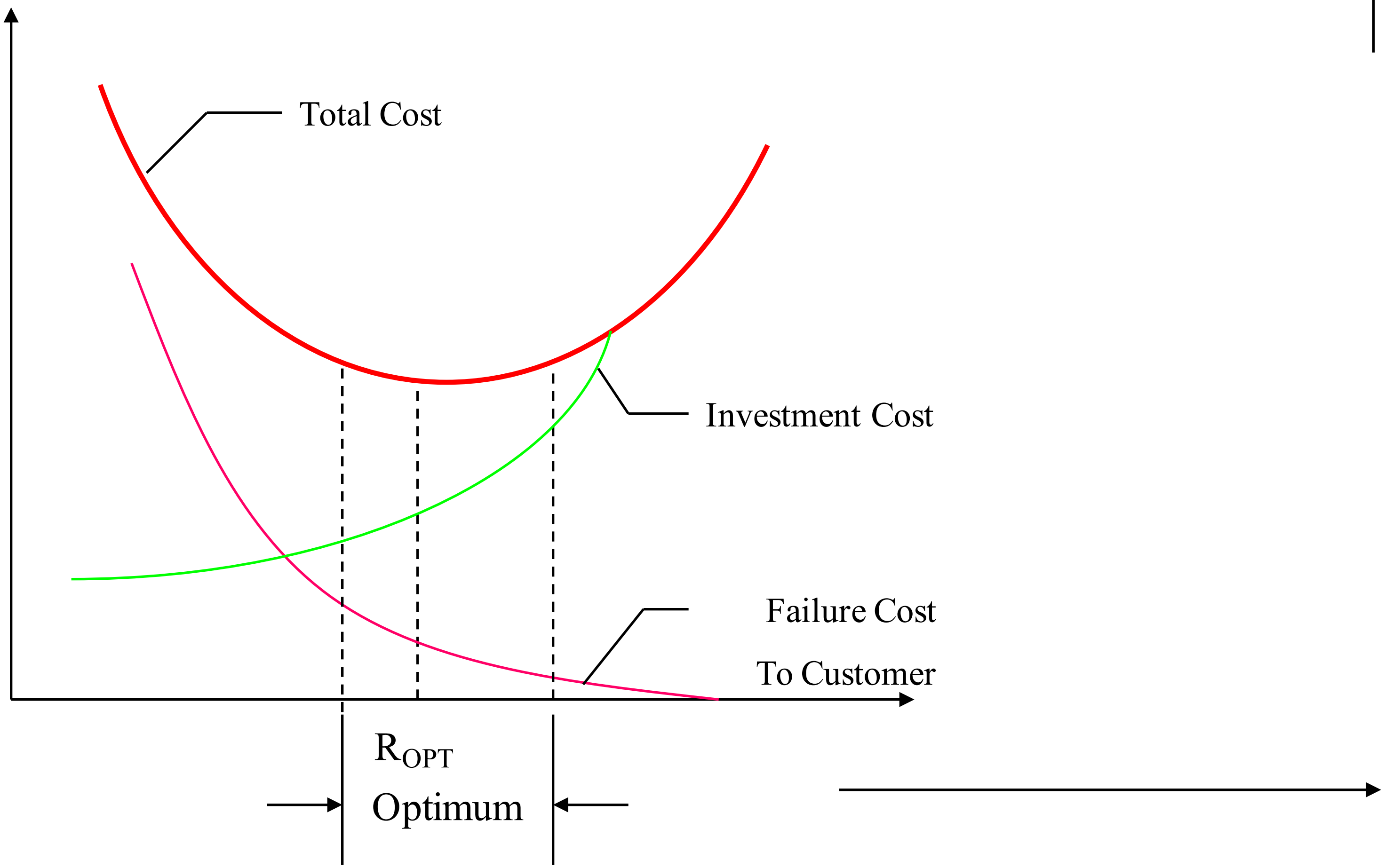
Estimate of mean up time(MUT) = 320/4=80 h

Estimate of mean cycle time= 86.25 h=MUT+MDT=1/FF

Basic Approaches to Reliability Considerations

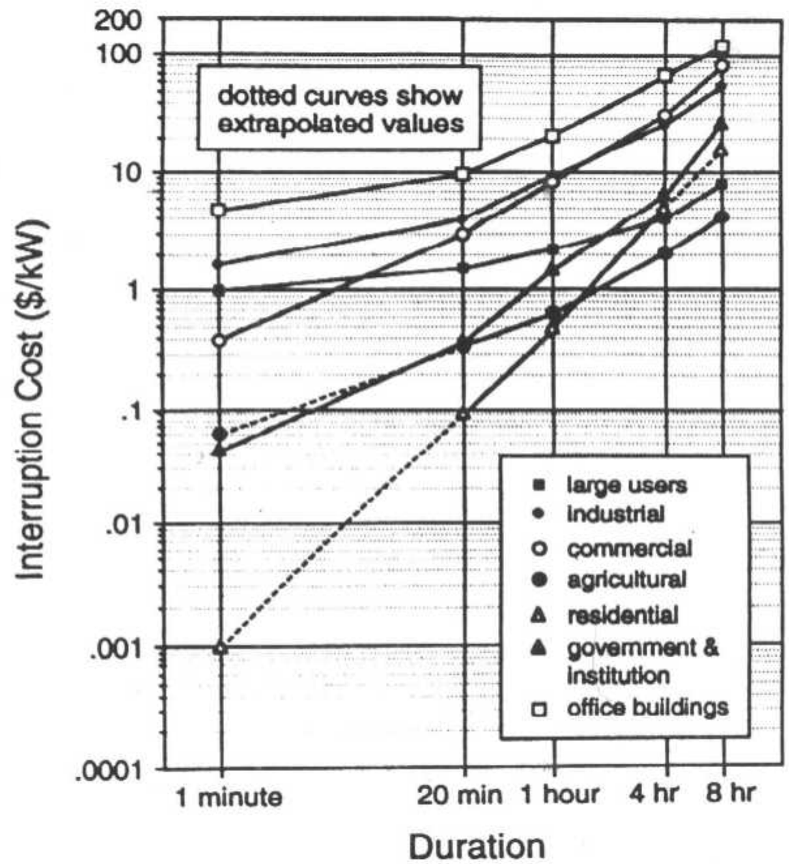
- Reliability implemented as a constraint
- Reliability worth included in overall cost optimization
- Multi-objective optimization with reliability as one of the objectives

Cost VS Reliability



Reliability Cost/Worth Indices

- Reliability worth assessment provides the opportunity to incorporate cost analysis in system design.
- Two approaches:
  - As an objective: Achieve minimum total cost, which is the sum of investment, operating and customer interruption/failure costs.
  - As constraint: Expected power interruption or cost associated with failure is less than a pre-selected value.



Interruption / Failure Cost

$$IC = \sum_{i=1}^{n} L_i \cdot \lambda_i \cdot c_i(d_i)$$

n : the number of load points
L_i : load requirement [kW]
λ_i : failure frequency [f/year]
$c_i(d_i)$: customer damage function [$/kW]
d_i : outage duration [hours].

Multi-objective Optimization & Pareto-optimality

- Most problems in nature have several (very often conflicting) objectives to be satisfied or optimized.
- **Multi-objective optimization** (or programming) also known as **multi-criteria** or **multi-attribute** optimization, is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints.
- Standard definition

  Min $\mathbf{f} = [f_1(x), f_2(x), \ldots, f_n(x)]$

  subject to $x \in S$ (constraints)

  where each $f_1(x)$ is an objective function
- One solution often employed is to optimize a weighted objective function

  Min $\boldsymbol{f} = [w_1 f_1(x) + w_2 f_2(x), \ldots + w_n f_n(x)]$

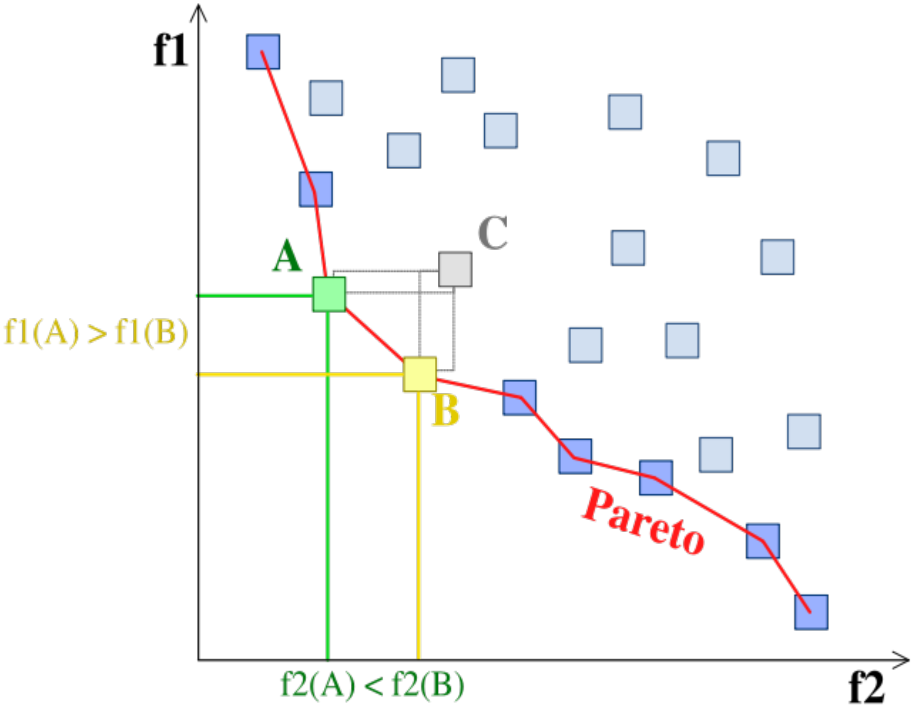
  subject to $x \in S$ (constraints)

  where each $f_1(x)$ is an objective function
- An other technique is to pick one objective as the primary objective to be optimized and transform the remaining objectives into constraints.

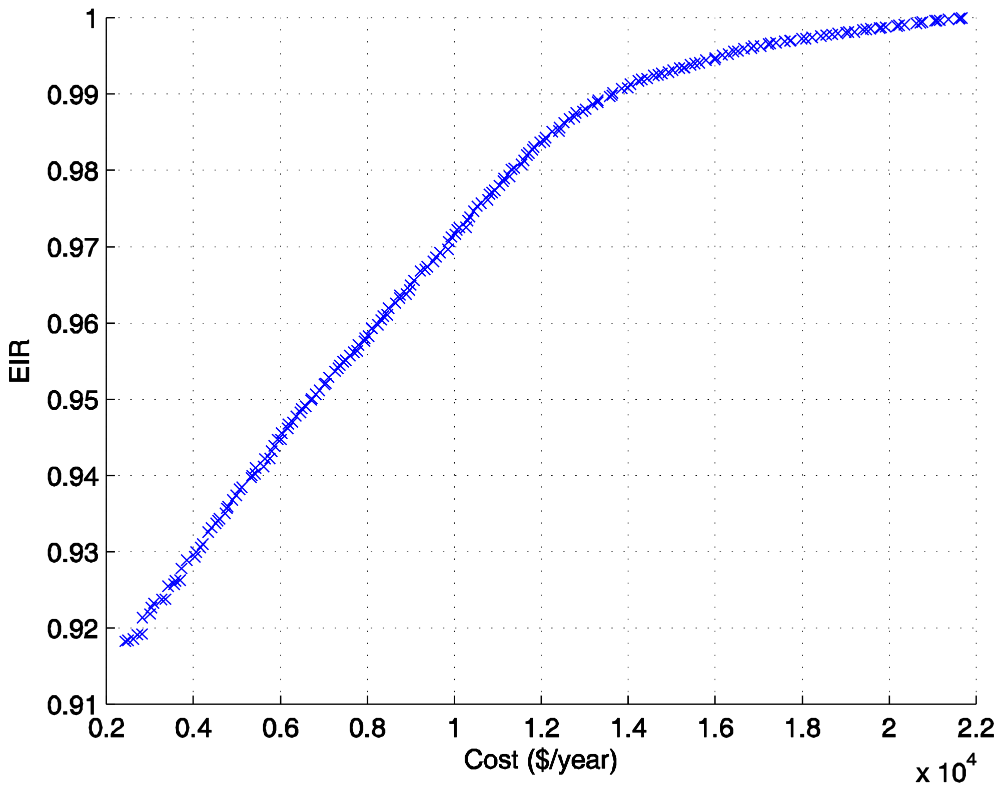
- In multi-objective optimization an other approach is to find Pareto-optimal solutions.
- No part of Pareto optimal solution can be improved without making some other part worse.
- This approach is useful when it is difficult to formulate a global objective function.

Pareto Frontier

- See opposite the objective space of f_1 and f_2
- Given that lower values are preferred to higher values, point ***C*** is not on the Pareto Frontier because it is dominated by both point *A* and point *B*; and Points *A* and *B* are non-inferior.
- A vector is said to be dominated if other vectors of system variables can be found that have improved values of any function f_i without creating a lower value in any objectives in **f**.



An Example: Tradeoff Curve Between Reliability & Cost



Question Reformulated

- WE HAVE A LOAD OF 500 MW
- WHAT IS THE PROBABILITY NOT SUPPLYING THE LOAD(loss of load) IN THE FOLLOWING SCENARIOS ?

  - 5 GENERATORS 100 MW EACH

  - 6 GENERATORS OF 100 MW EACH

  -12 GENERATORS OF 50 MW EACH
- ASSUME PROBABILITY OF FAILURE OF EACH GENERATOR IS 0.1 IN ALL CASES.

References:

1. C. Singh & R. Billinton, System Reliability Modelling and Evaluation, Hutchinson, London, 1977. (you can download from my website)
2. J. Endrenyi, Reliability Modeling in Electric Power Systems, John Wiley, 1978
3. R. Billinton & R. Allan, Reliability Evaluation of Power Systems, Plenum Press, 1984
4. Course Notes by C. Singh
5. Selected papers