Module 3
Frequency Balance Approach
for
System Reliability Analysis

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Concept of Transition Rate

- Consider two system states i and j.
- Transition rate from state i to j is the mean number of transitions from state i to j per unit of time in state i.

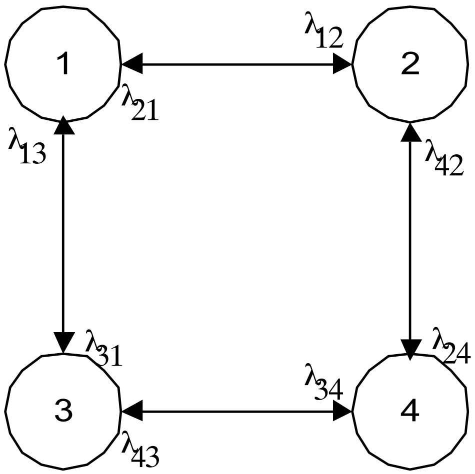
Concept of Transition Rate

- If the system is observed for T hours and Ti hours are spent in state i, then the transition rate from state i to j is given by

$$\lambda_{ij} = n_{ij}/T_i$$

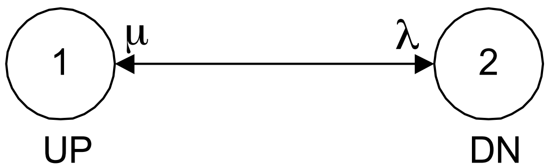
Where

n_{ij} = number of transitions from state i to j during the period of observation



Four state model

Example 1 : A 2- state component



A 2-state model

- Let UP state be #1 and Down state be #2.

- Transition rate from up to down state = failure rate

= n12 / T1 = 1 / (T1 / n12)= 1/ MUT

where

MUT= mean up time

- Transition rate from down to up state = repair rate

= n21 / T2 = 1/(T2 / n21)=1 / MDT

where

MDT = mean down time of the component

Concept of Frequency

- Frequency of encountering state j from state i is the expected (mean) number of transitions from state i to state j per unit time.
- $Fr(i \rightarrow j)$ = steady state or average frequency of transition from state i to j

$= n_{ij} / T$

$= (T_i / T)(n_{ij} / T_i)$

$= p_i \lambda_{ij}$

- where

p_i = long term fraction of time spent in state i

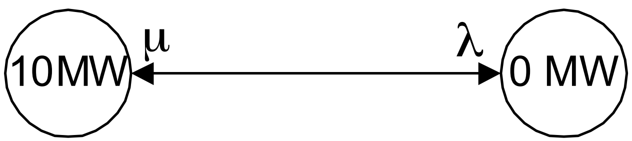
= steady state probability of system state i

Concept of Frequency Balance

- In steady state or average behavior,

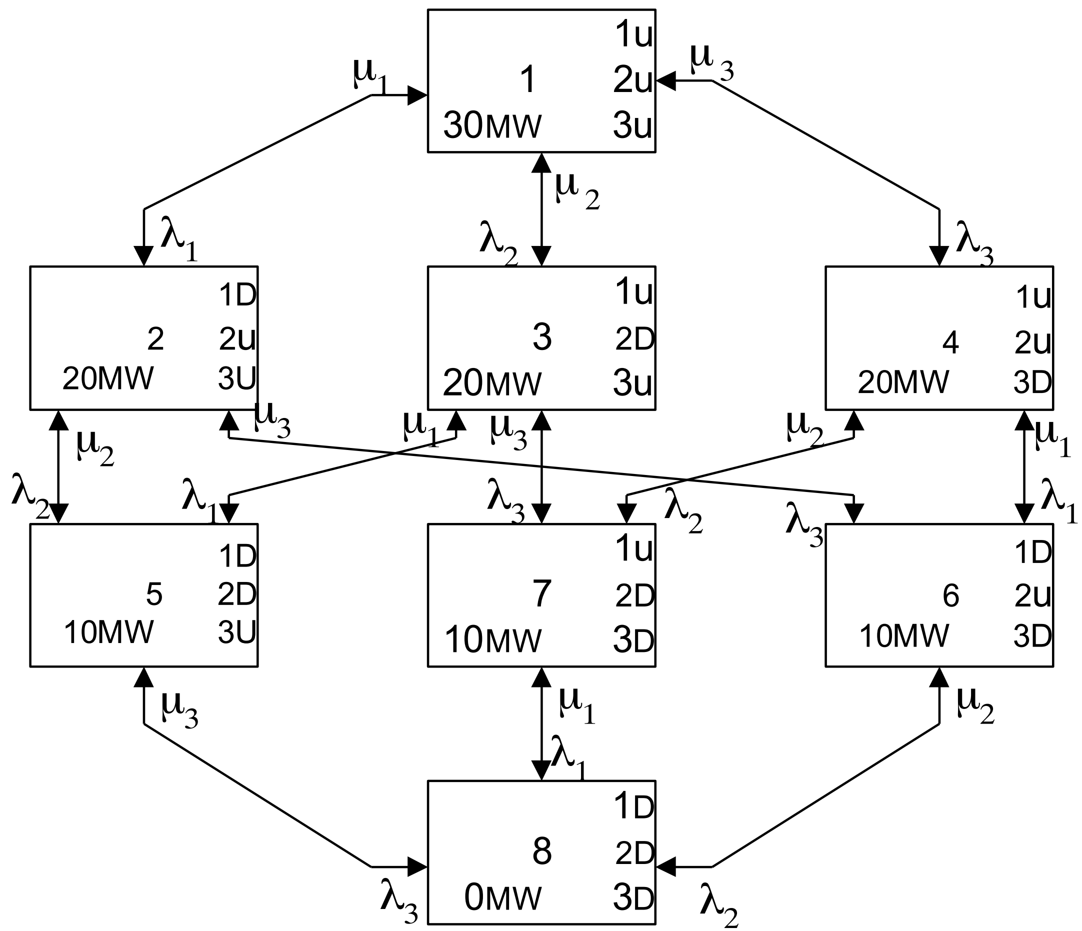
- Frequency of encountering a state (or a subset of states) equals the frequency of exiting from the state (or the subset of states).

Example 2



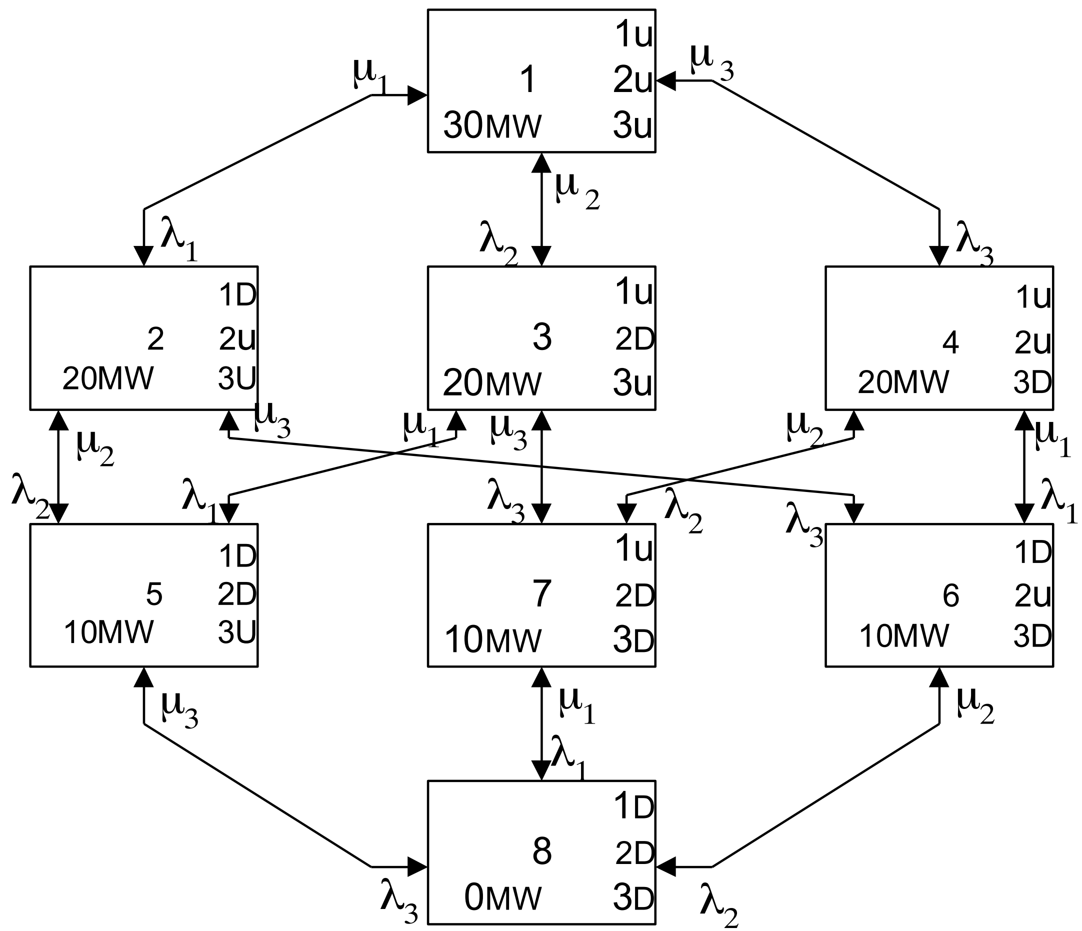
2 state model for a single unit

- Consider the state transition diagram for three 2-state units.



Example 2

- Equation for state 2 can be written as,
- $p_1\lambda_1 + p_5\mu_2 + p_6\mu_3 = p_2 \ (\mu_1 + \lambda_2 + \lambda_3)$



Calculation of state probabilities

- If components are independent, system state probabilities can be found by the product of unit state probabilities.

- If components are not independent then

  - write an equation for each of n system states.

  - any n-1 equations together with

$$\sum_{i=1}^{n} P_i = 1$$

  - can be solved to find state probabilities.

Calculation of state probabilities

- Note that
- Mean cycle time of an event (MCT) = 1 / frequency of the event
- Mean duration of the event = MCT x Prob. of the event

- Thus
- Mean cycle time between failures = 1/ freq of failure
- Mean down time (MDT) = prob of failure / freq of failure
- Mean up time = MCT - MDT

  = prob of system up / freq of failure

Problems to be solved and discussed

- Problem 1

Prove that for a two state unit,

p1 =probability of up state

$= \mu / (\lambda + \mu)$

and

p2 = probability of down state

$= \lambda / (\lambda + \mu)$

where

λ = failure rate of the component

and

μ = repair rate of the component

Problems to be solved and discussed

- Problem 2

A system consists of two identical transmission lines each capable of supplying full load. The failure rate of each line is 10 per year and the repair time is 10 hours.

1. Draw the state transition diagram of the system
2. Calculate
   (i) probability of system failure,
   (ii) frequency of system failure,
   (iii) mean up time and mean down time.

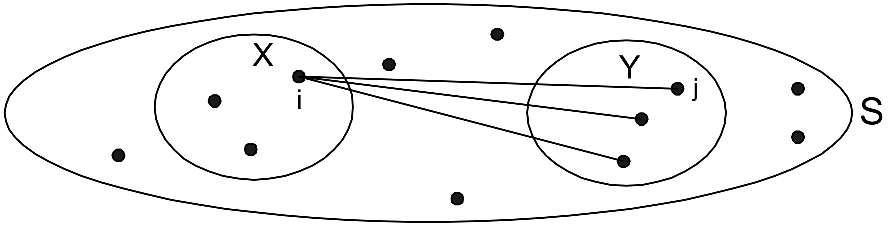
Frequency of a Set of States

Frequency of encountering subset Y from subset X,

$$Fr(X \rightarrow Y) = \sum_{i \in X} p_i \sum_{j \in Y} \lambda_{ij}$$

Therefore freq of encountering subset X,

$$Fr(X) = \sum_{i \in (S-X)} p_i \sum_{j \in X} \lambda_{ij}$$

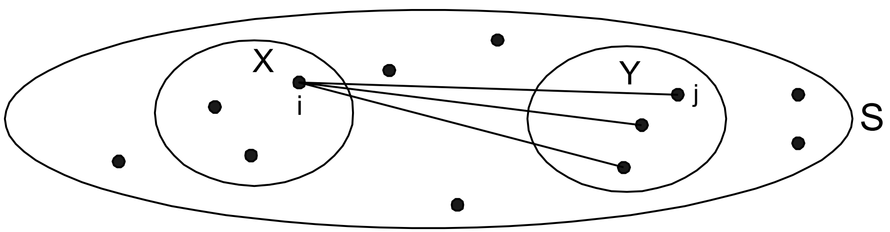


Frequency of a Set of States

To find the frequency of a subset of states:

1. Draw boundary around the subset.

2. Find the expected transition rate into the boundary or out of the boundary.

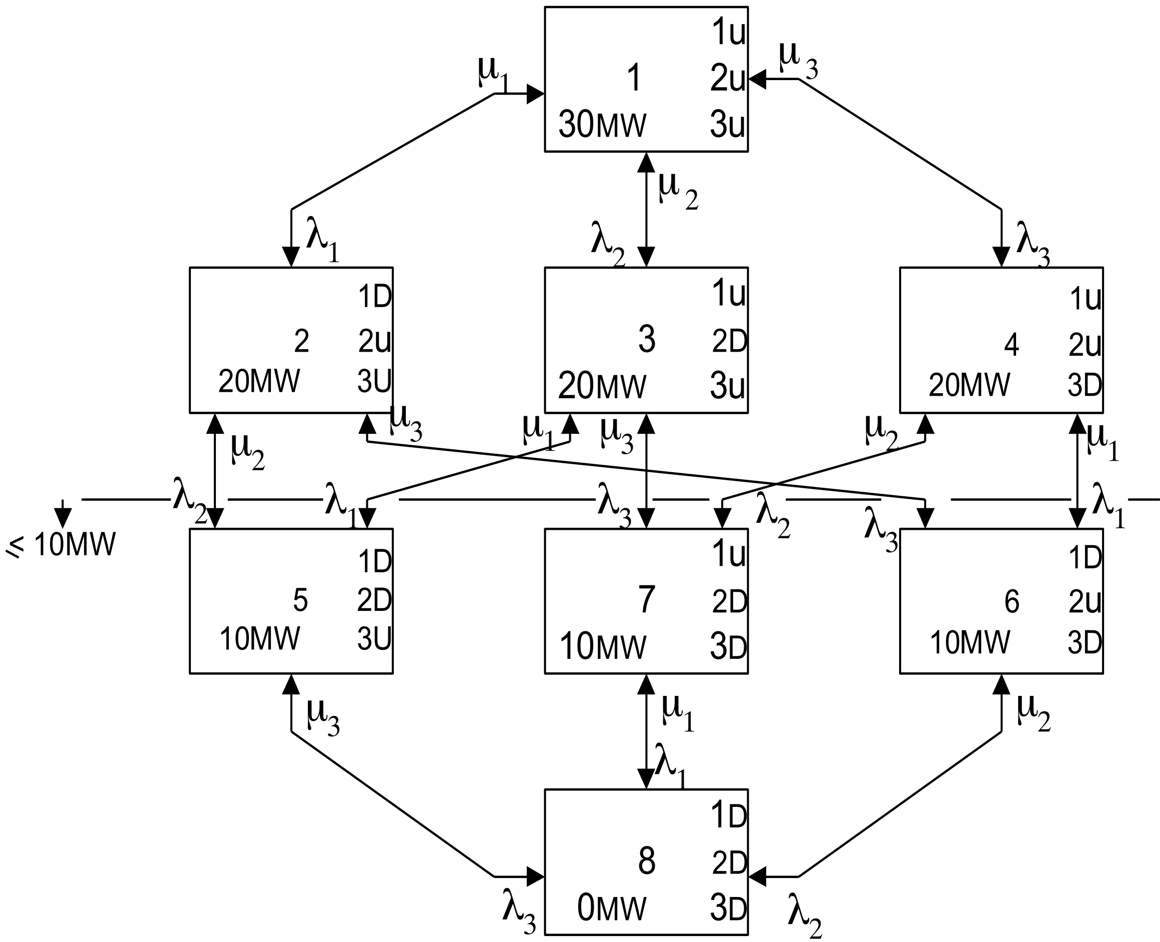


Example

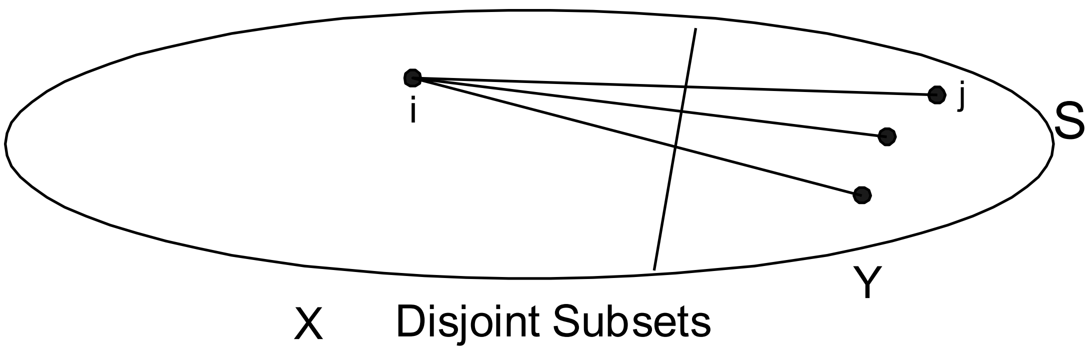
For the case of three 2-state units,

Fr(capacity $\leq$ 10)
$= p_2 (\lambda_2 + \lambda_3) + p_3 (\lambda_1 + \lambda_3) + p_4 (\lambda_1 + \lambda_2)$
$= p_5(\mu_1 + \mu_2) + p_7 (\mu_2 + \mu_3) + p_6 (\mu_1 + \mu_3)$

This frequency is typically called cumulative frequency.



Equivalent Transition Rate



The equivalent transition rate from X to Y in above figure is given by

$$\lambda_{XY} = Fr(X \rightarrow Y) / Prob(X)$$

Problems to be solved and discussed

Problem 3

Draw a 4-state state transition diagram of a system consisting of two identical components, each having a failure rate of λ and repair rate of μ.

1. Reduce this diagram to a 3 state diagram using the concept of equivalent transition rate.
2. Find the probability and frequency of both components down.

Problem 4

Solve problem 3 assuming that only one line can be repaired at a time.