Module 4-2
Methods of Quantitative Reliability Analysis

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METHODS OF QUANTITATIVE RELIABILITY ANALYSIS

- ANALYTICAL METHODS
  - STATE SPACE USING MARKOV PROCESSES
  - NETWORK REDUCTION
  - MIN CUT SETS

- MONTE CARLO SIMULATION
  - NONSEQUENTAL - RANDOM SAMPLING
  - TIME SEQUENTIAL

- CONCEPT OF RELIABILITY COHERENCE
EXAMPLE SYSTEM

Generator

Transmission

Load

B0

B1

B2

B3

B4
EXAMPLE SYSTEM

- **Generators:**
  Each generator either has full capacity of 50 MW or 0 MW when failed. Failure rate of each generator is 0.1/day and mean-repair-time is 12 hours.

- **Transmission Lines:**
  The failure rate of each transmission line is assumed to be 10 f/y during the normal weather and 100 f/y during the adverse weather. The mean down time is 8 hours. Capacity of each line is 100 MW.

- **Weather:**
  The weather fluctuates between normal and adverse state with mean duration of normal state 200 hours and that of adverse state 6 hours.

- **Breakers:**
  Breakers are assumed perfectly reliable except that the pair B1&B2 or B3&B4 may not open on fault on the transmission line with probability 0.1.

- **Load:**
  Load fluctuates between two states, 140 MW and 50 MW with mean duration in each state of 8hr and 16hr respectively.
EXAMPLE SYSTEM

FOR THE DESCRIBED SYSTEM, HOW CAN YOU CALCULATE THE FOLLOWING BASIC RELIABILITY INDICES?

1. Loss of load probability
2. Frequency of loss of load
3. Mean duration of loss of load
Solving Example Problem using Markov Approach

- 1. System State Description & Equivalents

- The first task is to obtain probabilities for the generators, transmission lines and loads, which are independent parts of the system.

- 1.1. Generators

Each Generator has two possible states:

\[
\mu = \frac{1}{12} = \frac{730}{8760} \text{ / year} \quad \lambda = 0.1 / \text{day} = 36.5 / \text{year}
\]
Solving Example Problem using Markov Approach

State Transition Diagram – Generator System
Solving Example Problem using Markov Approach

- Merging Identical Capacity States

**Equivalent State Transition Diagram – Generator System**

- Equivalent transition rates:

\[
\lambda_{12G} = \frac{P_1 \lambda + P_1 \lambda + P_1 \lambda}{P_1} = 3\lambda \\
\mu_{21G} = \frac{P_2 \mu + P_3 \mu + P_4 \mu}{P_2 + P_3 + P_4} = \mu \\
\lambda_{23G} = \frac{2P_2 \lambda + 2P_3 \lambda + 2P_4 \lambda}{P_2 + P_3 + P_4} = 2\lambda \\
\mu_{32G} = \frac{2P_5 \mu + 2P_6 \mu + 2P_7 \mu}{P_5 + P_6 + P_7} = 2\mu \\
\lambda_{34G} = \frac{P_5 \lambda + P_6 \lambda + P_7 \lambda}{P_5 + P_6 + P_7} = \lambda \\
\mu_{43G} = \frac{P_8 \mu + P_8 \mu + P_8 \mu}{P_8} = 3\mu
\]
Solving Example Problem using Markov Approach

- Transition rate matrix is:

\[
R_G = \begin{bmatrix}
-3\lambda & 3\lambda & 0 & 0 \\
\mu & -(\mu + 2\lambda) & 2\lambda & 0 \\
0 & 2\mu & -(2\mu + \lambda) & \lambda \\
0 & 0 & 3\mu & -3\mu \\
\end{bmatrix}
\]

- If we substitute values for \( \mu \) and \( \lambda \) obtained in the beginning into above matrix, transition rate matrix for the generator system is:

\[
R_G = \begin{bmatrix}
-109.5 & 109.5 & 0 & 0 \\
730 & -803 & 73 & 0 \\
0 & 1460 & -1496.5 & 36.5 \\
0 & 0 & 2190 & -2190 \\
\end{bmatrix}
\]
Solving Example Problem using Markov Approach

- 1.2. Transmission Lines

Each Transmission line has two possible states

- During the normal weather \( \lambda = 10 / \text{year} \)
- During the adverse weather \( \lambda' = 100 / \text{year} \)

\[
\mu = \frac{1}{8} = 1095 / \text{year}
\]
Solving Example Problem using Markov Approach

- If all the breakers are perfectly reliable, for the two-transmission-line system, there will be 4 states.
Solving Example Problem using Markov Approach

If breakers may not open on command:

![Diagram showing a six-state transition diagram for a transmission system. The states and transitions are labeled with probabilities and states such as State 1: 1U 2U 100 MW, State 2: 1D 2U 100 MW, State 3: 1U 2D 100 MW, State 4: 1D 2D 0 MW, State 5: 1D 2U 0 MW, State 6: 1U 2D 0 MW. Probabilities include 0.1λ and 0.9λ.]
Solving Example Problem using Markov Approach

- Merging of states:

![Diagram of Four State Transition Diagram - Transmission System]

- Equivalent transition rates:

\[
\lambda_{12} = \frac{P_5 \mu + P_6 \mu}{P_5 + P_6} = \mu \\
\lambda_{23} = \frac{P_1 (0.9 \lambda + 0.9 \lambda)}{P_1} = 1.8 \lambda \\
\lambda_{34} = \frac{P_2 \lambda + P_3 \lambda}{P_2 + P_3} = \lambda
\]

\[
\mu_{21} = \frac{P_1 (0.1 \lambda + 0.1 \lambda)}{P_1} = 0.2 \lambda \\
\mu_{32} = \frac{P_2 \mu + P_3 \mu}{P_2 + P_3} = \mu \\
\mu_{43} = \frac{P_4 (\mu + \mu)}{P_4} = 2 \mu
\]
Solving Example Problem using Markov Approach

- 1.2.1 Weather

Transition rate from normal weather to adverse weather is:

\[ N = \frac{1}{200} \times \frac{8760}{8760} = 43.8 \text{ / year} \]

Transition rate from adverse weather to normal weather is:

\[ S = \frac{1}{6} \times \frac{8760}{8760} = 1460 \text{ / year} \]
Solving Example Problem using Markov Approach

- Transition rate matrix of transmission system is:

\[
R_T = \begin{bmatrix}
-(2\lambda + N) & 1.8\lambda & 0 & 0.2\lambda & N & 0 & 0 & 0 \\
\mu & -(\mu + \lambda + N) & \lambda & 0 & 0 & N & 0 & 0 \\
0 & 2\mu & -(2\mu + N) & 0 & 0 & 0 & N & 0 \\
\mu & 0 & 0 & -(\mu + N) & 0 & 0 & 0 & N \\
S & 0 & 0 & 0 & -(2\lambda' + S) & 1.8\lambda' & 0 & 0.2\lambda' \\
0 & S & 0 & 0 & \mu & -(\mu + \lambda'S) & \lambda' & 0 \\
0 & 0 & S & 0 & 0 & 2\mu & -(2\mu + S) & 0 \\
0 & 0 & 0 & S & \mu & 0 & 0 & -(\mu + S)
\end{bmatrix}
\]
Solving Example Problem using Markov Approach

- 1.3 Load

State Transition Diagram – Load

\[
\begin{align*}
\lambda_{21} &= \frac{1}{16} = 547.5 \text{ / year} \\
\lambda_{12} &= \frac{1}{8} = 1095 \text{ / year}
\end{align*}
\]

- Transition rate matrix:

\[
R_L = \begin{bmatrix}
-1095 & 1095 \\
547.5 & -547.5
\end{bmatrix}
\]
Solving Example Problem using Markov Approach

- 2 Steady State Probabilities, Frequency and Mean Duration of Loss of Load
- 2. 1. Generation System
- In order to get the steady probability of each state, we can write:

\[
\begin{bmatrix}
P_{1G} \\
P_{2G} \\
P_{3G} \\
P_{4G}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
R^T_G
\]

\[
\sum_{i=1}^{4} P_{iG} = 1
\]

- Using the RG we obtained to solve above equation, we get the steady state probability of each state.
- If generators are independent probabilities can be calculated by product rule also.
- Probabilities calculated in either way are the same.

\[
P_u = \frac{\mu}{\lambda + \mu} = 0.95238
\]

\[
P_d = \frac{\lambda}{\lambda + \mu} = 0.047619
\]

\[
P_{1G} = P_u \times P_u \times P_u = 0.8638377
\]

\[
P_{2G} = 3 \times P_u \times P_u \times P_d = 0.1295725
\]

\[
P_{3G} = 3 \times P_u \times P_d \times P_d = 0.00647876
\]

\[
P_{4G} = P_d \times P_d \times P_d = 0.000107979
\]
Solving Example Problem using Markov Approach

- 2.2. Transmission System
- We have the following equations:

\[
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4 \\
P_5 \\
P_6 \\
P_7 \\
P_8 \\
\end{bmatrix}^T \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} = \sum_{i=1}^{8} P_i = 1
\]
Solving Example Problem using Markov Approach

- Using the RT we obtained to solve previous equation, we get the steady state probability of each state:

\[
R_T^T = \omega
\]

\[
1.0e+003 * \omega
\]

\[
\begin{matrix}
-0.0638 & 1.0950 & 0 & 1.0950 & 1.4600 & 0 & 0 & 0 \\
0.0180 & -1.1488 & 2.1900 & 0 & 0 & 1.4600 & 0 & 0 \\
0 & 0.0100 & -2.2338 & 0 & 0 & 0 & 1.4600 & 0 \\
0.0020 & 0 & 0 & -1.1388 & 0 & 0 & 0 & 1.4600 \\
0.0438 & 0 & 0 & 0 & -1.6600 & 1.0950 & 0 & 0 \\
0 & 0.0438 & 0 & 0 & 0.1800 & -2.6550 & 2.1900 & 0 \\
0 & 0 & 0.0438 & 0 & 0 & 0.1000 & -3.6500 & 0 \\
0 & 0 & 0 & 0.0438 & 0.0200 & 0 & 0 & -2.5550
\end{matrix}
\]

\[
P_1=0.9507726
\]

\[
P_2=0.01787034
\]

\[
P_3=0.0001196528
\]

\[
P_4=0.0019820304
\]

\[
P_5=0.02678843
\]

\[
P_6=0.002162378
\]

\[
P_7=0.000060686
\]

\[
P_8=0.00024383
\]
Solving Example Problem using Markov Approach

- We can also reduce the eight-state transmission transition diagram to a three-state diagram with respect to the capacities of the states:

\[ \begin{align*}
\text{State 1} & : 200\text{MW} \\
\text{State 2} & : 100\text{MW} \\
\text{State 3} & : 0\text{MW}
\end{align*} \]
Solving Example Problem using Markov Approach

- For the reduced model, the following results apply:

\[ P_{1T} = P_1' + P_3' = 0.9507726 + 0.02678843 = 0.97756103 \]
\[ P_{2T} = P_2' + P_6' = 0.01787034 + 0.002162378 = 0.020032718 \]
\[ P_{3T} = P_3' + P_4' + P_7' + P_8' = 0.000119628 + 0.0019820304 + 0.000060686 + 0.00024383 = 0.00246192 \]

\[ \lambda_{12T} = \frac{P_1' \cdot 1.8 \lambda + P_3' \cdot 1.8 \lambda}{P_1' + P_3'} = \frac{0.9507726 \cdot 18 + 0.02678843 \cdot 180}{0.97756103} = 22.439339 \]

\[ \lambda_{23T} = \frac{P_2' \lambda + P_6' \lambda}{P_2' + P_6'} = \frac{0.01787034 \cdot 10 + 0.002162378 \cdot 100}{0.020032718} = 19.714808 \]

\[ \lambda_{13T} = \frac{P_1' \cdot 0.2 \lambda + P_3' \cdot 0.2 \lambda}{P_1' + P_3'} = \frac{0.9507726 \cdot 2 + 0.02678843 \cdot 20}{0.97756103} = 2.493259 \]

\[ \mu_{21T} = \frac{P_2' \mu + P_6' \mu}{P_2' + P_6'} = \mu = 10 \]

\[ \mu_{32T} = \frac{P_3' \cdot 2 \mu + P_7' \cdot 2 \mu}{P_3' + P_7' + P_4' + P_8'} = 1095 \]

\[ \mu_{31T} = \frac{P_4' \mu + P_8' \mu}{P_3' + P_7' + P_4' + P_8'} = 2090 \]
Solving Example Problem using Markov Approach

- 2.3. Load
- The following equations apply:

\[ R_L^T \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

\[ \sum_{i=1}^{2} P_{iL} = 1 \]

- Using the RL we obtained to solve above equations we get the steady state probability in each state:

\[ \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} 0.3333333 \\ 0.6666667 \end{bmatrix} \]
Solving Example Problem using Markov Approach

2.4. Solution for the System

Steady state probability, frequency and mean time of loss of load could be found using the following table:

\[
\begin{align*}
P_{1G} &= 0.8638377 \\
P_{2G} &= 0.1295725 \\
P_{3G} &= 0.00647876 \\
P_{4G} &= 0.000107979 \\
P_{1T} &= 0.97756103 \\
P_{2T} &= 0.020032718 \\
P_{3T} &= 0.0024061992 \\
P_{1L} &= 0.3333333 \\
P_{2L} &= 0.6666667
\end{align*}
\]
Solving Example Problem using Markov Approach

<table>
<thead>
<tr>
<th>System State</th>
<th>Generation, transmission, load system state</th>
<th>Probability of system state</th>
<th>Transition to the states with loss of load</th>
<th>Loss of load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111</td>
<td>0.281485</td>
<td>2, 3, 4 (\lambda_{12T}, \lambda_{13T}, \lambda_{12G})</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>121</td>
<td></td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>131</td>
<td></td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>211</td>
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<td>Yes</td>
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<td>6</td>
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<td>7</td>
<td>311</td>
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</tr>
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<td>8</td>
<td>321</td>
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</tr>
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<td>9</td>
<td>331</td>
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</tr>
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<td>411</td>
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</tr>
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<td>11</td>
<td>421</td>
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<td></td>
<td>Yes</td>
</tr>
<tr>
<td>12</td>
<td>431</td>
<td></td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>13</td>
<td>112</td>
<td>0.562969</td>
<td>15(\lambda_{12T})</td>
<td>No</td>
</tr>
<tr>
<td>14</td>
<td>122</td>
<td>0.0115367</td>
<td>2, 15 (\lambda_{12I}, \lambda_{12I})</td>
<td>No</td>
</tr>
<tr>
<td>15</td>
<td>132</td>
<td></td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>16</td>
<td>212</td>
<td>0.0844453</td>
<td>4, 18 (\lambda_{21T}, \lambda_{13T})</td>
<td>No</td>
</tr>
<tr>
<td>17</td>
<td>222</td>
<td>0.0017305</td>
<td>5, 18 (\lambda_{21I}, \lambda_{23I})</td>
<td>No</td>
</tr>
<tr>
<td>18</td>
<td>232</td>
<td></td>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>19</td>
<td>312</td>
<td>0.00422226</td>
<td>7, 21, 22, (\lambda_{21I}, \lambda_{13I}, \lambda_{12I})</td>
<td>No</td>
</tr>
<tr>
<td>20</td>
<td>322</td>
<td>0.000086525</td>
<td>8, 21, 23 (\lambda_{21I}, \lambda_{31I}, \lambda_{32I})</td>
<td>No</td>
</tr>
<tr>
<td>21</td>
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<td></td>
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</tr>
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</tr>
<tr>
<td>24</td>
<td>432</td>
<td></td>
<td></td>
<td>Yes</td>
</tr>
</tbody>
</table>
Solving Example Problem using Markov Approach

- We can calculate the probability of states having no load loss. Those probabilities are obtained for the generators, transmission lines and loads as independent.

- From previous Table, we can get the steady state probability of the loss of load as follows.

\[
P = 1 - (0.281485 + 0.562969 + 0.0115367 + 0.0844453 + 0.0017305 + 0.00422226 + 0.000086525) \\
P = 0.053524715
\]
Solving Example Problem using Markov Approach

- The frequency of loss of load is:

\[
F = \sum_{i \in X'} \sum_{j \in X'} P_{ij} \lambda_{ij} = P_1 (\lambda_{12T} + \lambda_{13T} + \lambda_{12G}) + P_{13} \lambda_{13T} + P_{14} (\lambda_{21L} + \lambda_{23T})
\]
\[
+ P_{16} (\lambda_{21L} + \lambda_{13T}) + P_{17} (\lambda_{21L} + \lambda_{23T}) + P_{19} (\lambda_{21L} + \lambda_{13T} + \lambda_{34G}) + P_{20} (\lambda_{21L} + \lambda_{23T} + \lambda_{34G})
\]

\[
F = 95.742635 / \text{year}
\]

- Values needed for F that are calculated previously:

\[
\lambda_{12T} = 22.439339 \quad \lambda_{23T} = 19.714808 \quad \lambda_{13T} = 2.493259
\]
\[
\lambda_{12G} = 3\lambda = 109.5
\]
\[
\lambda_{23G} = 2\lambda = 73
\]
\[
\lambda_{34G} = \lambda = 36.5
\]
\[
\lambda_{21L} = 547.5 / \text{year}
\]
Solving Example Problem using Markov Approach

- The mean time of loss of load is:

\[ MD = \frac{P}{F} = \frac{0.053524715}{95.742635/\text{year}} = 4.89726\text{hours} \]
Cut Set Method

- A cut set is a set of components or conditions that cause system failure.
  - A min cut set is a cut set that does not contain any cut set as a subset.
  - In this presentation a cut set implies a min cut set.
  - The term component will be used to indicate both a physical component as well as a condition.

- Components in a given cut set are in parallel, as they all need to fail to cause system failure.

- Cut sets are in series as any cut set can cause system failure.
Frequency & Duration Equations For Cut Sets

● First Order Cut Set: One component involved

\[ \lambda_{csk} = \lambda_i \quad r_{csk} = r_i \]

where

\[ \lambda_i, r_i = \text{Failure rate and mean duration of component } i \]

\[ \lambda_{csk}, r_{csk} = \text{Failure rate and mean duration of cut set } k \text{ that contains component } i \]
Frequency & Duration Equations
For Cut Sets

- Second Order Cut Set k: Two components involved

\[ \lambda_{csk} = \frac{\lambda_i \lambda_j (r_i + r_j)}{1 + \lambda_i r_i + \lambda_j r_j} \]
\[ r_{csk} = \frac{r_i r_j}{r_i + r_j} \]

where

\( \lambda_i, \lambda_j \) = Failure rates of components \( i \) and \( j \) comprising cut set \( k \),

\( r_i, r_j \) = Mean failure durations of components \( i \) and \( j \) comprising cut set \( k \).
Frequency & Duration Equations For Cut Sets

- Second Order Cut Set with Components subject to Normal and Adverse Weather.

\[ \lambda', \lambda = \text{Failure rate of component } i \text{ in the normal and adverse weather} \]

\[ N, S = \text{Mean duration of normal and adverse weather} \]

\[ \lambda_{ck} = \lambda_a + \lambda_b + \lambda_c + \lambda_d \]

\[ \lambda_a = \frac{N}{N+S} \left( \frac{\lambda_i \lambda_j \cdot N r_i}{N+r_i} + \frac{\lambda_j \lambda_i \cdot N r_j}{N+r_j} \right) \]

\[ \lambda_b = \frac{N}{N+S} \left( \lambda_i \frac{r_i}{N} \cdot \lambda_j \frac{S r_i}{S+r_i} + \lambda_j \frac{r_j}{N} \cdot \lambda_i \frac{S r_j}{S+r_j} \right) \]

\[ \lambda_c = \frac{S}{N+S} \left( \frac{\lambda_i \lambda_j \cdot N r_i}{N+r_i} + \lambda_j \lambda_i \frac{N r_j}{N+r_j} \right) \]

\[ \lambda_d = \frac{S}{N+S} \left( \lambda_i \frac{r_i}{N} \cdot \lambda_j \frac{S r_j}{S+r_j} + \lambda_j \frac{r_j}{N} \cdot \lambda_i \frac{S r_j}{S+r_j} \right) \]

\[ \lambda_a = \text{Component due to both failures occurring during normal weather} \]

\[ \lambda_b = \text{Initial failure in normal weather, second failure in adverse weather} \]

\[ \lambda_c = \text{Initial failure in adverse weather, second failure in normal weather} \]

\[ \lambda_d = \text{Both failures during adverse weather} \]
Combining n Cut Sets

\[ \lambda_T = \lambda_{cs1} + \lambda_{cs2} + \cdots + \lambda_{csn} \]

\[ r_T = \left( \lambda_{cs1} r_{cs1} + \lambda_{cs2} r_{cs2} + \cdots + \lambda_{csn} r_{csn} \right) / \lambda_T \]
APPLICATION OF CUT SET METHOD TO EXAMPLE SYSTEM

- Cut set 1: One line failure and breaker stuck.

\[
\lambda_{av} = \frac{\lambda_N + \lambda_S}{N + S} = 12.621 \\
\lambda_{c1} = 2 \times \lambda_{av} \times 0.1 = 2.524 \text{ f/y} \\
r_{c1} = 8 \text{ hr}
\]

- Cut set 2: One generator failure and load changes from 50 to 140

\[
\lambda_g = 0.1 \text{ / day} = 36.5 \text{ / year} \\
\lambda_{load} = \frac{8760}{16} = 547.5 \text{ / year} \\
r_g = 12 \text{ hr} = 0.00137 \text{ yr} \\
r_{load} = 8 \text{ hr} = 0.000913 \text{ yr} \\
\lambda_{c2} = \frac{\lambda_g \lambda_{load} (r_g + r_{load})}{(1 + \lambda_g r_g + \lambda_{load} r_{load})} = 88.306 \text{ / yr} \\
r_{c2} = \frac{r_g r_{load}}{r_g + r_{load}} = 4.8 \text{ hr}
\]
APPLICATION OF CUT SET METHOD TO EXAMPLE SYSTEM

- Cut set 3: One line failure (breaker not stuck) and load changes from 50 to 140.

  \[ \lambda_i = \lambda_{av} \times 0.9 \]
  \[ r_i = 8\text{hr} \]

  \[ \lambda_{cs3} = \frac{\lambda_i \lambda_{load} (r_i + r_{load})}{(1 + \lambda_i r_i + \lambda_{load} r_{load})} = 15.03/\text{yr} \]
  \[ r_{cs3} = \frac{r_i r_{load}}{r_i + r_{load}} = 4\text{hr} \]

- Cut set 4: Two lines fail (breaker not stuck)

  - For each line

  \[ \lambda = 10 \times 0.9 = 9/\text{yr} \]
  \[ \lambda' = 100 \times 0.9 = 90/\text{yr} \]
  \[ r = 8\text{hr} = 0.000913\text{yr} \]
  \[ N = 200\text{hr} = 0.022831\text{yr} \]
  \[ S = 6\text{hr} = 0.000685\text{yr} \]

- Applying the equation for second order cut set exposed to fluctuating environment,

  \[ \lambda_{cs4} = 0.3888/\text{yr} \]
  \[ r_{cs4} = 4\text{hr}. \]
APPLICATION OF CUT SET METHOD TO EXAMPLE SYSTEM

- Cut set 4: Two lines fail (breaker not stuck)
- For each line

  \[ \lambda = 10 \times 0.9 = 9 \text{ / yr} \]
  \[ \lambda' = 100 \times 0.9 = 90 \text{ / yr} \]
  \[ r = 8 \text{ hr} = 0.00913 \text{ yr} \]
  \[ N = 200 \text{ hr} = 0.022831 \text{ yr} \]
  \[ S = 6 \text{ hr} = 0.000685 \text{ yr} \]

- Applying the equation for second order cut set exposed to fluctuating environment,

  \[ \lambda_{cs4} = 0.3888 \text{ / yr} \]
  \[ r_{cs4} = 4 \text{ hr} \]

- For the system

  \[ \lambda_T = \lambda_{cs1} + \lambda_{cs2} + \lambda_{cs3} + \lambda_{cs4} = 106.25 \text{ / yr} \]
  \[ r_T = \frac{(\lambda_{cs1} r_{cs1} + \lambda_{cs2} r_{cs2} + \lambda_{cs3} r_{cs3} + \lambda_{cs4} r_{cs4})}{\lambda_T} = 4.76 \text{ hr} \]
  \[ \mu_T = \frac{1}{r_T} \]

  Frequency of failure \[ = \frac{\mu_T}{\lambda_T + \mu_T} \]
  \[ \lambda_T = 100.45 \text{ / yr} \]

  Probability of failure \[ = \frac{\lambda_T}{\lambda_T + \mu_T} \]
  \[ = 0.0546 \]