Module 6-3
Basic Concept of Continuous Distribution Approximation

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Continuous Distribution Approximation

- The approximation of generation system model by continuous distributions is based on the following basic ideas:

1. The capacity outage of the generation system, X, can be expressed as the sum of capacity outages of generating units, Xi being the capacity outage of unit i,

$$X = X_1 + X_2 + \ldots X_n$$

X_i = capacity outage of unit i

X = capacity outage of system

2. Cumulants of Xi can be found from the moments of Xi . Assuming units to be independent, cumulants of X can be found as the sum of cumulants of Xi .

Continuous Distribution Approximation

3. Moments of X can be found from cumulants of X.

4. Using the moments of X, a distribution can be fitted to X.

Approximation results because only a few moments are used in deriving the parameters of the series or distribution.

Moments and Cumulants

The j-th capacity outage moment of unit i,

$$\mu_{j,i} = \sum_{k=1}^{m} (Q_{k,i})^j \cdot p_{k,i}$$

Where

$Q_{k,i}$= capacity outage of unit i in state k

$P_{k,i}$= probability of state k of unit i.

Cumulants of unit i are given by

$$\kappa_{1,i} = \mu_{1,i}$$

$$\kappa_{j,i} = \mu_{j,i} - \sum_{r=1}^{j-1} \binom{j-1}{r} \mu_{r,i} \kappa_{j-r}, \quad j \geq 2$$

where

$$\kappa_{j,i} = j\text{-th order cumulant of unit } i$$

$\kappa_{j,i}$ = j-th order cumulant of unit i

Moments and Cumulants

The j-th order cumulant of the generation system with m units (all units are assumed to be statistically independent) is given by

$$\kappa_j = \sum_{i=1}^{m} \kappa_{j,i}$$

Capacity outage moments of generation system are given by

$$\begin{aligned} m_1 &= \kappa_1 \\ m_j &= \kappa_j + \sum_{r=1}^{j-1} \binom{j-1}{r} m_r \kappa_{j-r}, \quad j \geq 2 \end{aligned}$$

Gram-Charlier Series

The calculation of LOLE consists of the following steps :

1. Calculate moments and cumulants of generating units from the given data.
2. Calculate cumulants of capacity outage of the system.
3. Probability density function of generating system capacity outage can then be given by the GC series as follows:

$$f(x)=g(x)\,[\,1- K_3 D^3/6 + K_4 D^4/24 - K_5 D^5/120 + (K_6+10K_3^2)D^6/720 + \ldots]$$

$$g(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

σ_i = standard deviation for unit i

$$K_j = \kappa_j / \left(\sum_{i=1}^{m} \sigma_i^2\right)^{j/2}$$

$$D^n = d^n/dx^n$$

Gram-Charlier Series

4. Probability of capacity outage equal or greater than X,

$$P_g(X) = 1 - \int_{-\infty}^{X} f(x)dx$$

5. The LOLE can be found either using Pg(X) or finding the cumulants of the load duration curve and then determining the pdf for the combination of generation and load.

For complete description, please see

http://www.ece.tamu.edu/People/bios/singh/coursenotes/part4.pdf