Module 7-1
Multi-Area
Reliability Analysis

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Problem Formulation

- In the traditional single area model, intra-area transmission constraints are ignored. Interpretted in another way, transmission lines are assumed to be capable of transferring power from generation to load points without any problem.

- In multi-area model inter-area transmission constraints are considered. The intra-area constraints are only indirectly considered since they impact the inter-area tie capacity.

- The single and multi-area models can be thus depicted as in the figures of next page:

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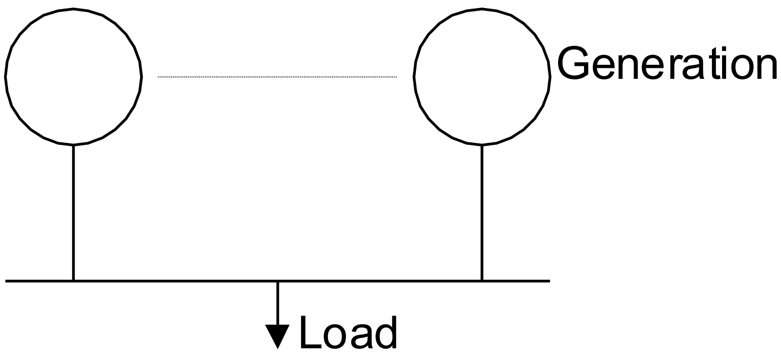
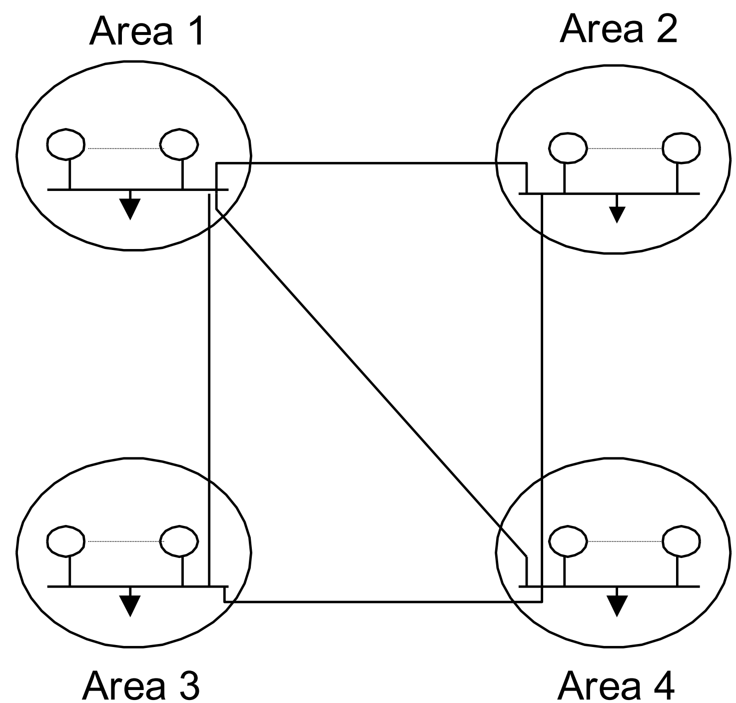
Problem Formulation



Traditional single area model



A typical multi-area model

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Problem Formulation

1. There are a number of generators in each area. These generators can be represented by various capacity outage states with appropriate probabilities.
2. The load in each area is fixed at a particular value.
3. The tie lines are assumed to have different capacity states with corresponding probabilities. The admittance of tie lines are not considered.
4. In case of loss of load in a given area, other areas give assistance so long as the do not lose their own load. This is called NLLS (No Load Loss Sharing) policy.

- The problem is to calculate reliability indices (LOLE and EUE) of each area and of the entire system.

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Problem Formulation

Other issues to be included :

1. Include hourly variation of load in each area, keeping in mind the correlation between area loads.
2. Include firm interchanges.
3. Consider jointly owned units.
4. Consider dependence of tie capacity on status of certain units.
5. Consider DC load flow model, including tie line admittance.

We will first discuss the basic model with looped connections. Then we will discuss inclusion of other feature and subsequently we will discuss the simple solution for radial connections.

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Multi-Area System Representation

- For simplicity, we will discuss the problem using an example of three areas since this will capture all the essential features of the problems and the solution techniques.

- The techniques are, however, general and applicable to many interconnected areas.

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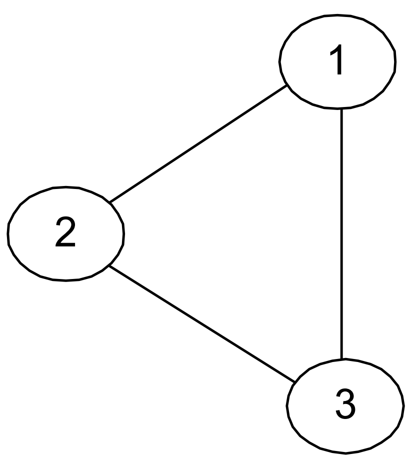
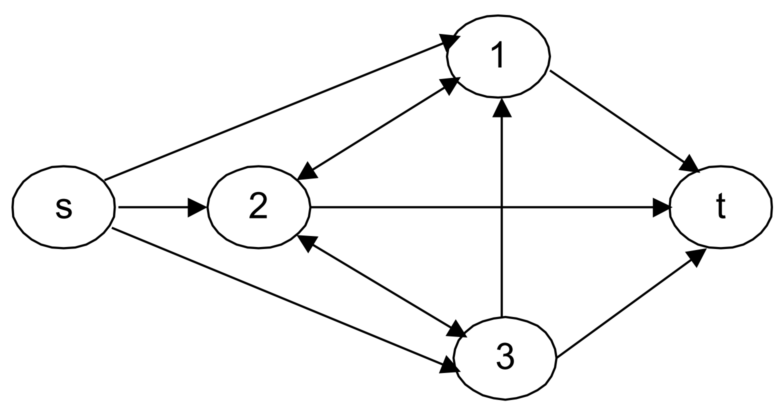
Multi-Area System Representation

- Consider the left configuration of three areas connected together.
- This three area configuration can be represented by a probabilistic flow network with capacitated arcs as in the right figure



A three area system



Flow network for three area system

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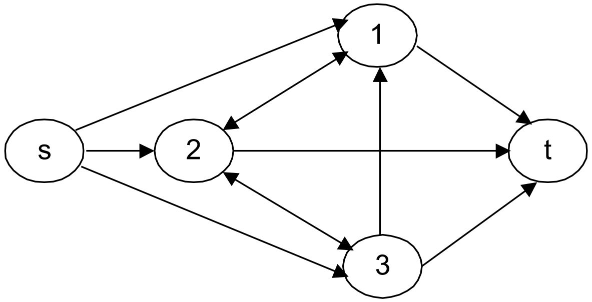
Multi-Area System Representation



- In this network, s stands for source node and t stands for terminal node. These nodes are introduced for purposes of calculation. The various arcs are as follows:

1. Generation arc: A directed arc from source node s to area i represents the discrete capacity states of generation in area i. The generation unit models in area i can be combined using unit addition algorithm to give the capacity states and corresponding probabilities.

2. Transmission arc: A bidirectional arc between nodes i and j represents the discrete capacity states of the tie line between areas i and j. These tie line states have their corresponding probabilities.

3. Load arc: A directed arc between node i and the terminal node t represents the load in area j. For the time being we will assume these loads to be fixed.

Multi-Area System States

- Capacity of arc i denoted by random variable Ci

$$P_{ij} = Prob\{C_i = C_{ij}\}$$

C_{ij} : Capacity value of arc i in state j.

- When each random variable ci takes a value, say ,this corresponds to a system state

$$x = (x_1, x_2, \dots \dots \dots x_n)$$

- The probability associated with state x

$$P(x) = \prod_{i=1}^{n} P_{i x_i}$$

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Multi-Area System States

- Example: Consider that areas 1,2 and 3 have capacities and probabilities given as follows:

Table 1 Capacities and cumulative probabilities for three area example

Area 1		Area 2		Area 3	
Cap	Cum Prob	Cap	Cum Prob	Cap	Cum Prob
		600	1.0		
500	1.0	500	.737856	500	1.0
400	.67232	400	.344640	400	.67232
300	.26272	300	.098880	300	.26272
200	.05792	200	.016960	200	.05792
100	.00672	100	.001600	100	.00672
0	.00032	0	.000064	0	.00032

Each transmission line has two states 100 MW (prob=.99) and 0 MW (prob=.01)

The loads in areas are

Area 1	Area 2	Area 3
400	500	400

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Multi-Area System States

These capacities can be arranged in the following tabular form :

Arc State	Capacities for arcs					
	1	2	3	4	5	6
7		600				
6	500	500	500			
5	400	400	400			
4	300	300	300			
3	200	200	200			
2	100	100	100	100	100	100
1	0	0	0	0	0	0

Number of states in this example is

6x7x6x2x2x2 = 2016

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Enumeration Approach

- Straight Forward Enumeration Approach Would Be:

1. Make a flow calculation for each state
2. Find the areas which have loss of load and the magnitude of loss of load.
3. Sum probabilities to find:

• Loss of Load Expectation (LOLE) is the mean number of loss of load hours/year.

• Expected Unserved Energy (EUE) is energy not supplied as a consequence of loss of load.

4. This approach is impractical for real systems because of extremely large number of states

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Flow Calculation

- Although in simple networks maximam flow could be calculated by inspection, in more complex networks Ford-Fulkerson algorithm is used.

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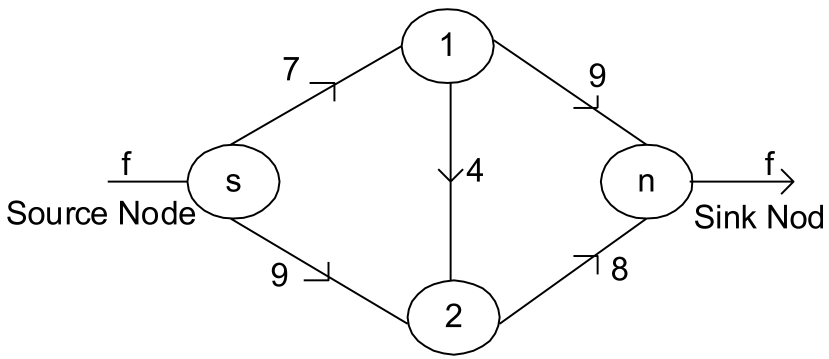
“Max-Flow Min-Cut Algorithm”

- This approach honors Kirchoff’s first law only.
- Example of a flow network with directed arc capacities is shown below.



Definitions

- Forward Arcs: The arcs leaving a node are called forward arcs with respect to that node. The arcs entering a node are backward arcs for that node.
- Path:A sequence of arcs starting from the source node and ending at the sink node
- Cycle:A path whose beginning and end are the same.

"Max-Flow Min-Cut Algorithm"

- Let N = Set of all nodes in the network.

$(H, \bar{H})$= A cut separating the source and sink is a partition of N into H and $\bar{H}$ such that $s \in H$ and $\mathrm{n} \in \bar{H}$.

- $C(H, \bar{H})$ = Capacity of cut is the sum of all the capacities of the arcs from the nodes in H to those in $\bar{H}$.
- Minimal Cut = The cut with the smallest capacity.

- Max-Flow Min-Cut Theorem [Ford and Fulkerson]:
- For any network, the value of the maximal flow from source to sink is equal to the capacity of the minimal cut.

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"Max-Flow Min-Cut Algorithm"

- Labeling Routine: Used to find a flow augmenting path from source to sink.
- Starting with s, node j can be labeled if a positive flow can be sent from s to j.
- From node i, any node j can be labeled if,
  - if the arc from i to j is a forward arc and flow in this arc is less than its capacity.

  or

  - if arc from i to j is a backward arc and the flow in the arc is greater than zero.
- If the labeling routine is continued until sink node is reached, then a flow augmenting path has been found.

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Max-Flow Algorithm

1. Start with a feasible flow on all arcs. Capacity restrictions and flow conservation at nodes must be satisfied.
2. Using labeling routine, find a flow augmenting path s to n through which a positive flow can be sent.
3. Compute the maximal flow δ that can be sent along the path.
4. Increase the flow on all forward arcs on the path by δ .
5. Find another flow augmenting path and repeat the procedure.

The algorithm terminates when no flow augmenting path can be found.

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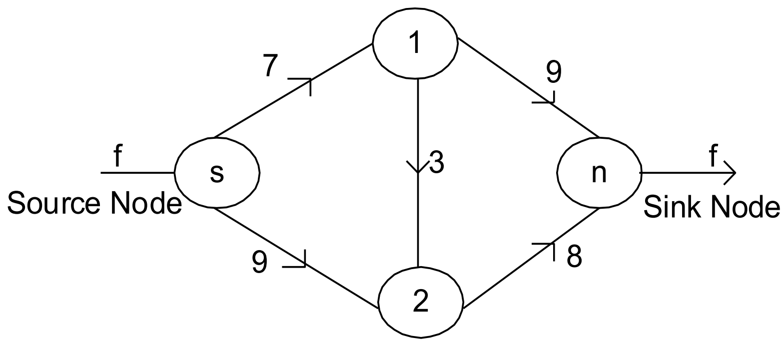
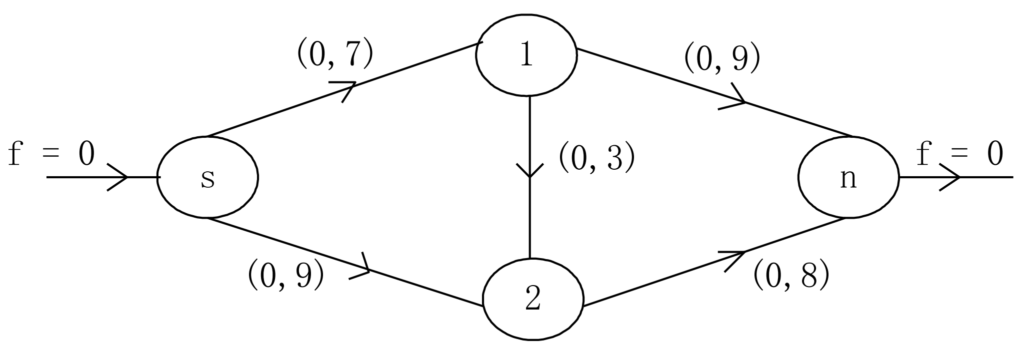
Example of Labeling Algorithm

The flow network is shown below:



Initial flows



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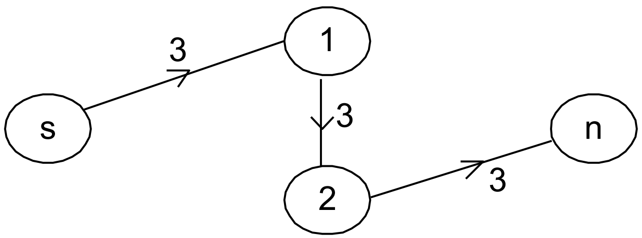
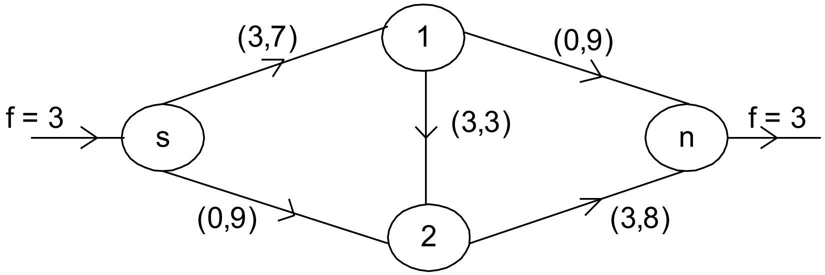
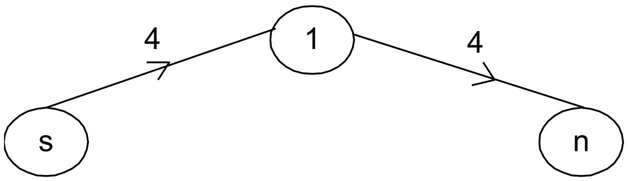
Example of Labeling Algorithm

Flows and augmentation paths:







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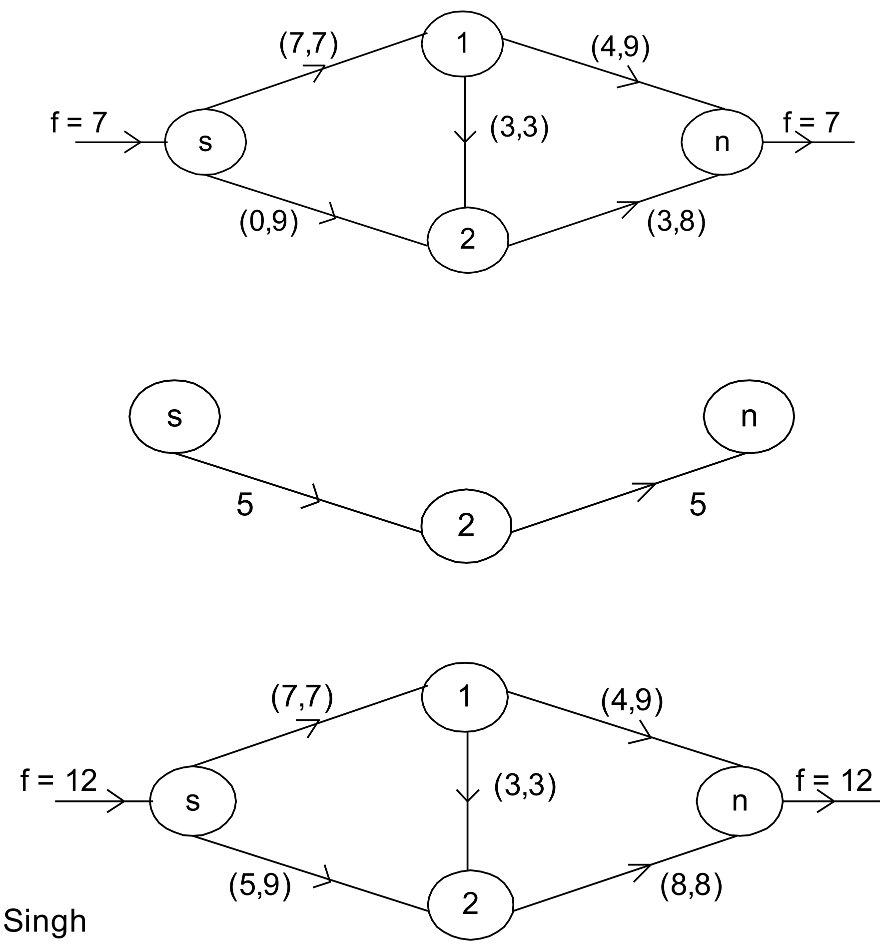
Example of Labeling Algorithm

Flows and augmentation paths:



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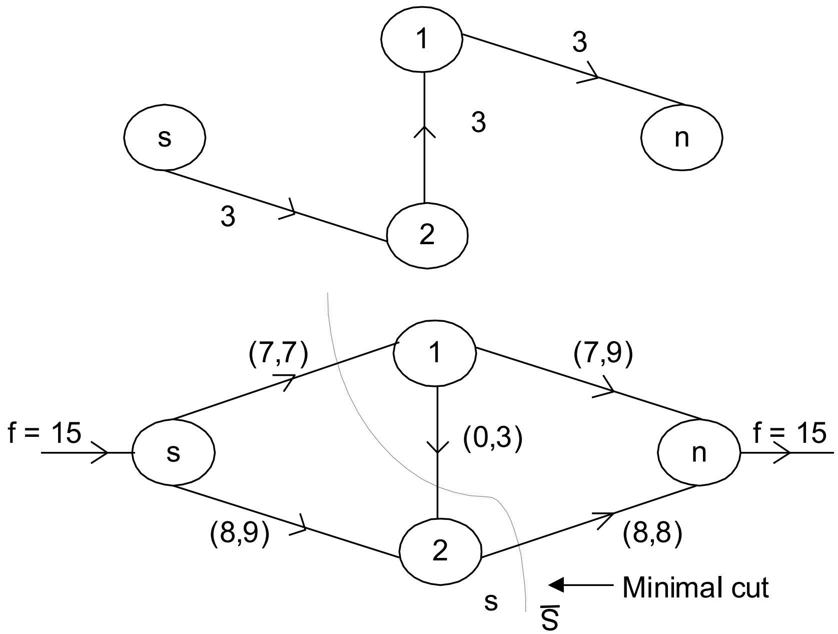
Example of Labeling Algorithm

Flows and augmentation paths:



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Solution Approaches

- It can be seen that the problem with the straightforward enumeration approach is that of dimensionality.
- The solution techniques attempt to overcome this problem by either arranging these states into sets or dealing with only a limited number of states and drawing inference from these states.
- These methods can be broadly classified into the following categories:

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Solution Approaches

1. Analytical methods:

These methods basically try to treat groups of states instead of states individually. This could be by either identifying cutsets or by identifying subsets of states. We will describe the decomposition method which decomposes the state space into sets.

2. Monte Carlo sampling:

Here states are sampled from the state space and indices are computed by statistical inference.

3. Hybrid methods:

Here a combination of analytical and simulation methods is employed. We will discuss Decomposition- Simlation Approach.

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