Module 2-1 Review of Probability Theory

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Outline

- Definitions
- Combinatorial probability rules
- Application to power system reliability

Definitions

- Sample space or state space
- Events
  - Union of events
  - Intersection of events
  - Disjoint events
  - Complement of an event
  - Independent events
- Probability of an event
- Conditional probability

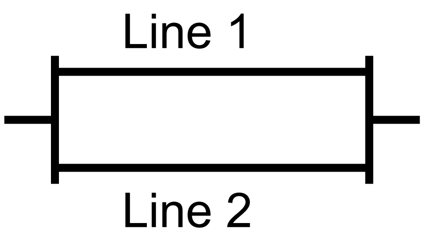
Sample Space or State Space

- The set of all possible outcomes of a random phenomenon

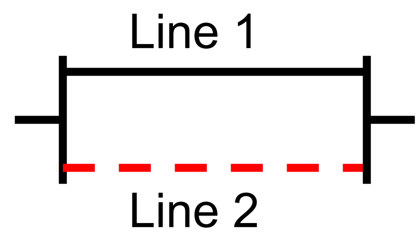
Example

- A status of a generator,
- state space = {Up, Down}

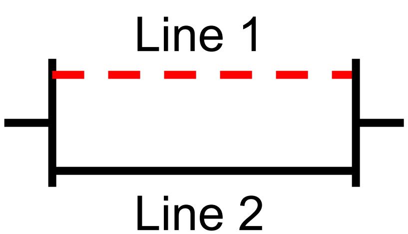
A Status of Two Transmission Lines



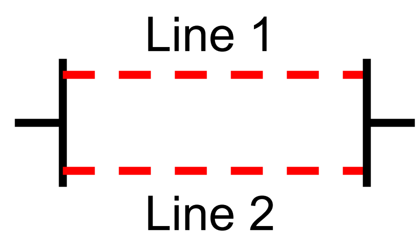
- (1 Up, 2 Up)



(1 Up, 2 Down)



(1 Down, 2 Up)



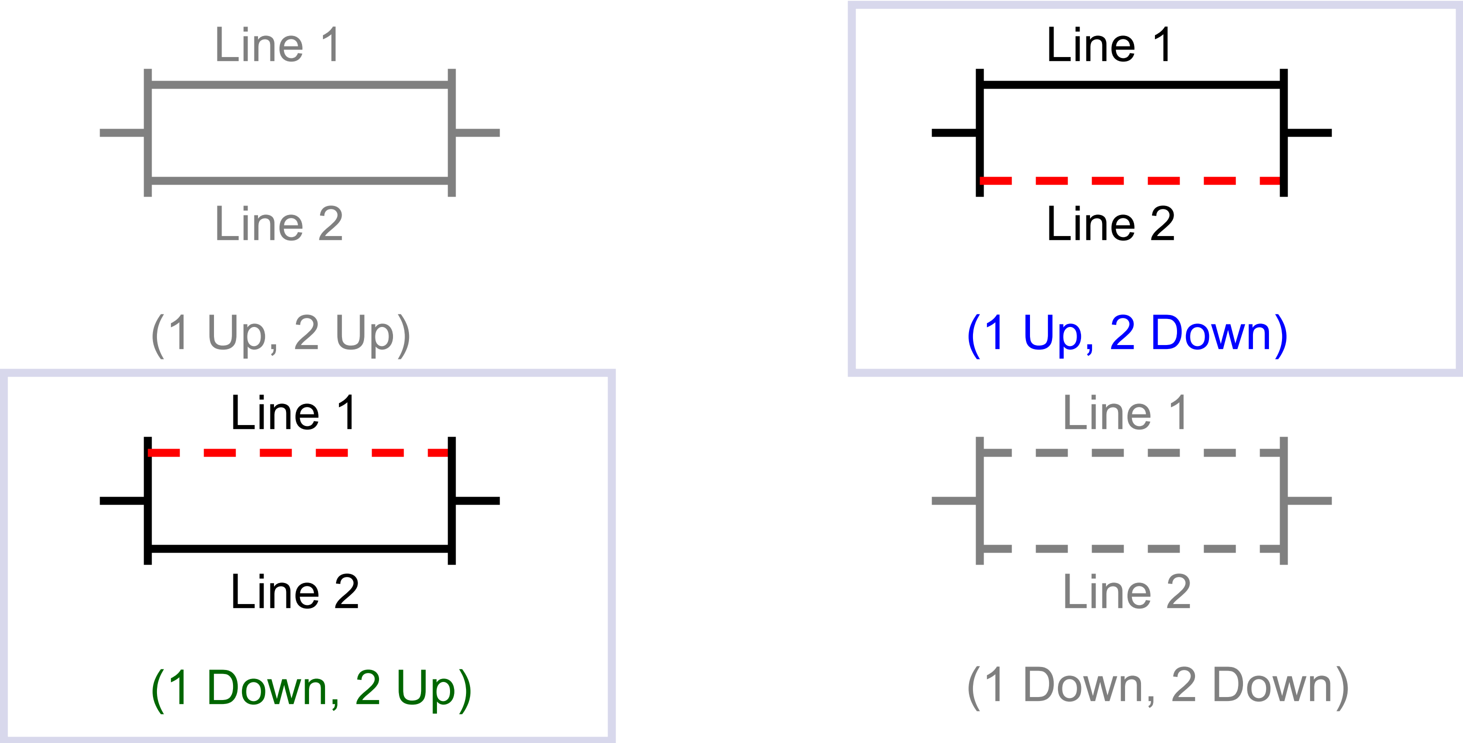
(1 Down, 2 Down)

State space = { (1U,2U), (1U,2D) , (1D,2U) , (1D,2D) }

Events

- A set of outcomes or a subset of a sample space.
- Example
  - Event of a generator fails, E = {Down}

An Event of One Transmission Line Fails



$$E = \{\ (1U,2D)\ ,\ (1D,2U)\ \}$$

Union of Events

- Union of event E_1 and E2 (E1 U E2) contains outcomes from either E1 or E2 or both.
- Examples
  - E_1 is an event that at least one line is up,

    E_1 = { (1U,2U), (1U,2D) , (1D,2U)}
  - E_2 is an event that at least one line is down,

    E_2 = { (1U,2D) , (1D,2U) , (1D,2D) }
  - Then, union of event E_1 and E_2 is,

    E = E_1 U E_2 = { (1U,2U), (1U,2D) , (1D,2U) , (1D,2D) }

Intersection of Events

- Intersection of event E_1 and E2 (E1∩E2) contains outcomes from both E1 and E2.
- Example
  - E_1 is an event that at least one line is up,

    $E_1 = \{ (1U,2U), (1U,2D), (1D,2U) \}$
  - E_2 is an event that at least one line is down,

    $E_2 = \{ (1U,2D), (1D,2U), (1D,2D) \}$
  - Then, intersection of event E_1 and E_2 is,

    $E = E_1 \cap E_2 = \{ (1U,2D), (1D,2U) \}$

    Which is that only one line is up

Disjoint Events

- Events that can not happen together.
- Example
  - E_1 is an event that two lines are up,

    $E_1 = \{ (1U,2U)\}$
  - E_2 is an event that two lines are down,

    $E_2 = \{ (1D,2D) \}$
  - Then, E_1 and E_2 are disjoint events.

Complement of an Event

- The set of outcomes that are not included in an event.
- Example
  - E_1 is an event that two lines are up,

$$E_1 = \{ (1U,2U)\}$$

  - $\bar{E}_1$ is a complement of E_1,

$$\bar{E}_1 = \{ (1U,2D) , (1D,2U) , (1D,2D) \}$$

Independent Events

- Events that happen independently.
- Example
  - Failure of a generator and failure of a line
  - Failure of line 1 and failure of line 2

Probability of an Event

- Number of times that an event occurs divided by total number of occurrences
- Properties,
  - $0 \le P(E) \le 1$
  - P(Impossible event) = 0
  - P(Sure event) = 1

Conditional Probability

- The probability of E2 given E1, P(E2 | E1), is the probability that event E2 occurs given that E1 has already occurred.
- If E_2 and E_1 are independent, then

$$P(E_2 \mid E_1) = P(E_2).$$

Probability Rules

- Combinatorial properties
  - Addition rule
  - Multiplication rule
  - Conditional probability rule
  - Complementation rule

Addition Rule

- A method of finding a probability of union of two events: Prob of E_1 or E_2 or both.

$$P(E_1 \cup E_2) = P(E1) + P(E2) - P(E1 \cap E2)$$

- If E_2 and E_1 are mutually exclusive, then

$$P(E_2 \cup E_1) = P(E_1) + P(E_2).$$

Multiplication Rule

- A method of finding a probability of intersection of two events: prob of E_1 and E_2.

$$P(E_1 \cap E_2) = P(E1) \times P(E2 \mid E1)$$

- If E1 and E2 are independent, then

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2).$$

Conditional Probability Rule

- If an event E depends on a number of mutually exclusive events B_j, then

$$P(E) = \sum_j [P(E|B_j) * P(B_j)]$$

Complementation Rule

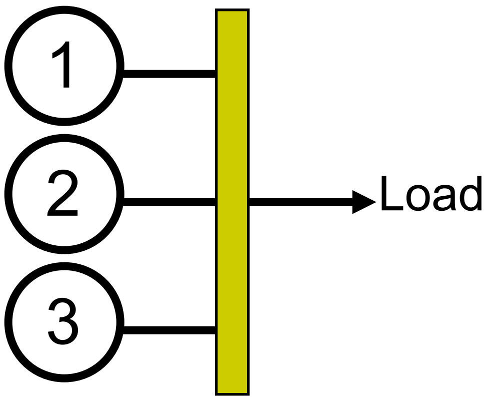
- Probability of the set of outcomes that are not included in an event.

$$P(\bar{E}) = 1 - P(E)$$

- Example

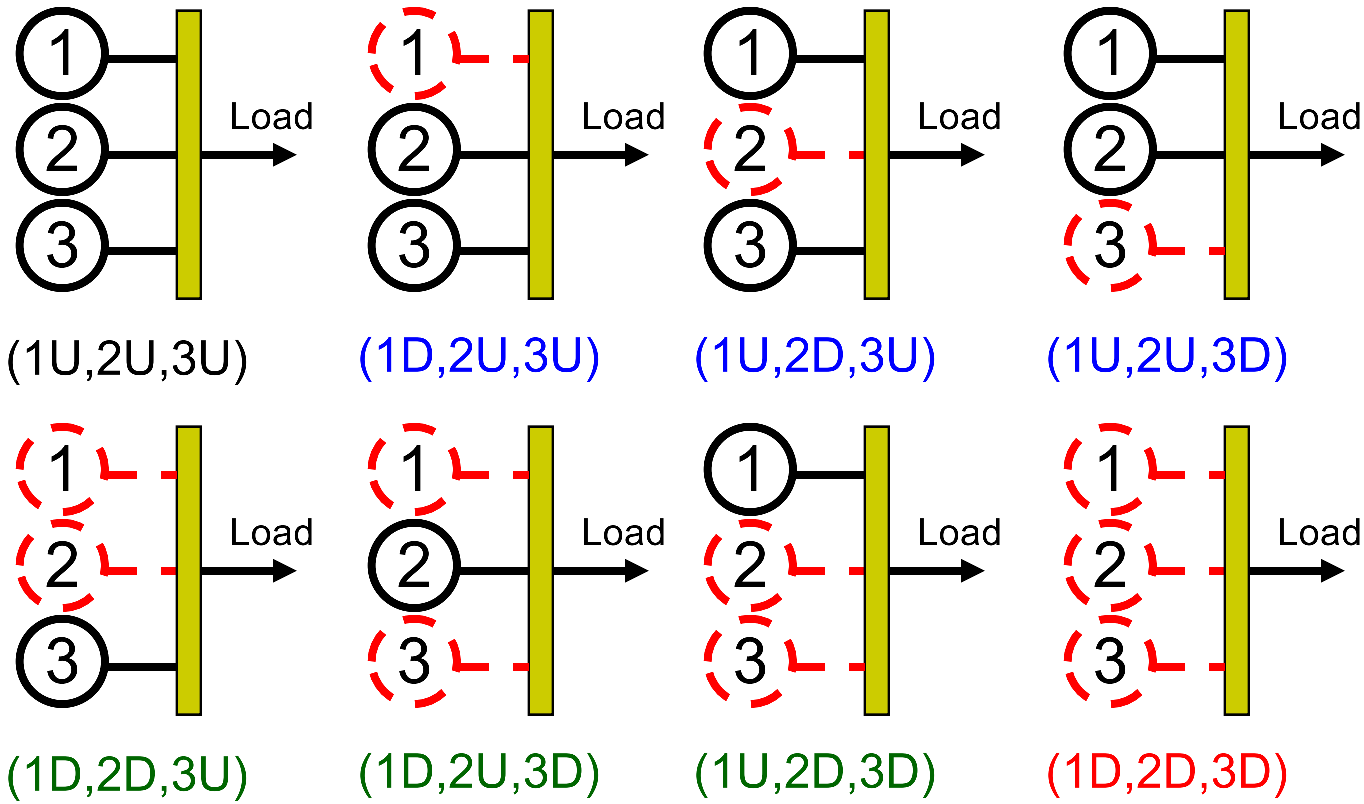
  Probability of success = 1 – Probability of failure

Example of Application to Calculation of Loss Of Load Probability - LOLP



- 3 generators
- Each with capacity 50 MW
- Identical probability of failure = 0.01
- Assume that each generator fails and is repaired independently.
- Find probability distribution of generating capacity.

State Space



State space = {(1U,2U,3U),(1D,2U,3U),(1U,2D,3U),(1U,2U,3D),(1D,2D,3U),(1D,2U,3D),(1U,2D,3D), (1D,2D,3D)}

Generating Capacity Probability Distribution

- Find a probability associated with each generating capacity levels
- 4 capacity levels, 0 MW, 50 MW, 100 MW, and 150 MW
- Let
  - E_0 be an event that generating capacity is 0 MW
  - E_1 be an event that generating capacity is 50 MW
  - E_2 be an event that generating capacity is 100 MW
  - E_3 be an event that generating capacity is 150 MW

Capacity 50 MW

- $E_1 = \{(1D,2D,3U),(1D,2U,3D),(1U,2D,3D)\}$

$$P(E_1) = P\{(1D,2D,3U) \cup (1D,2U,3D) \cup (1U,2D,3D)\}$$

- Using *addition rule*,

$$P(E_1) = P\{(1D,2D,3U)\} + P\{(1D,2U,3D)\} + P\{(1U,2D,3D)\}$$

$$P(E_1) = P(1D \cap 2D \cap 3U) + P(1D \cap 2U \cap 3D) + P(1U \cap 2D \cap 3D)$$

- Using *multiplication rule*,

$$P(E_1) = P(1D)\times P(2D)\times P(3U) + P(1D)\times P(2U)\times P(3D) + P(1U)\times P(2D)\times P(3D)$$

- Using *complementation*,

$$P(1U) = 1 - P(1D) = 1 - 0.01 = 0.99$$

- Then, $P(E_1) = 0.000297$

Capacity 100 MW

- $E_2 = \{(1D,2U,3U),(1U,2D,3U),(1U,2U,3D)\}$

  $$P(E_2) = P\{(1D,2U,3U) \cup (1U,2D,3U) \cup (1U,2U,3D)\}$$

- Using *addition rule*,

  $$P(E_2) = P\{(1D,2U,3U)\} + P\{(1U,2D,3U)\} + P\{(1U,2U,3D)\}$$

  $$P(E_2) = P(1D \cap 2U \cap 3U) + P(1U \cap 2D \cap 3U) + P(1U \cap 2U \cap 3D)$$

- Using *multiplication rule*,

  $$P(E_2) = P(1D)\times P(2U)\times P(3U) + P(1U)\times P(2D)\times P(3U) + P(1U)\times P(2U)\times P(3D)$$

- Using *complementation*,

  $$P(1U) = 1 - P(1D) = 1 - 0.01 = 0.99$$

- Then, $P(E_2) = 0.029403$

Capacity 150 MW

- $E_3 = \{(1U, 2U, 3U)\}$

$$P(E_3) = P(1U \cap 2U \cap 3U)$$

- Using *complementation*,

$$P(1U) = 1 - P(1D) = 1 - 0.01 = 0.99$$

- Using *multiplication rule*,

$$P(E_3) = P(1U) \times P(2U) \times P(3U)$$

$$P(E_3) = 0.99 \times 0.99 \times 0.99 = 0.970299$$

Generating Probability Distribution

Capacity (MW)	Probability
0	0.000001
50	0.000297
100	0.029403
150	0.970299

This is an example of probability density function or probability mass function.
Left column: values of discrete random variable ‘capacity’
Right column contains corresponding probabilities.

Loss of Load Probability

- Assume that load has the following distribution, find loss of load probability.
- Let E be the event that system suffers from loss of load, then

E = {Generation < Load}

Load (MW)	Probability
50	0.20
100	0.75
150	0.05

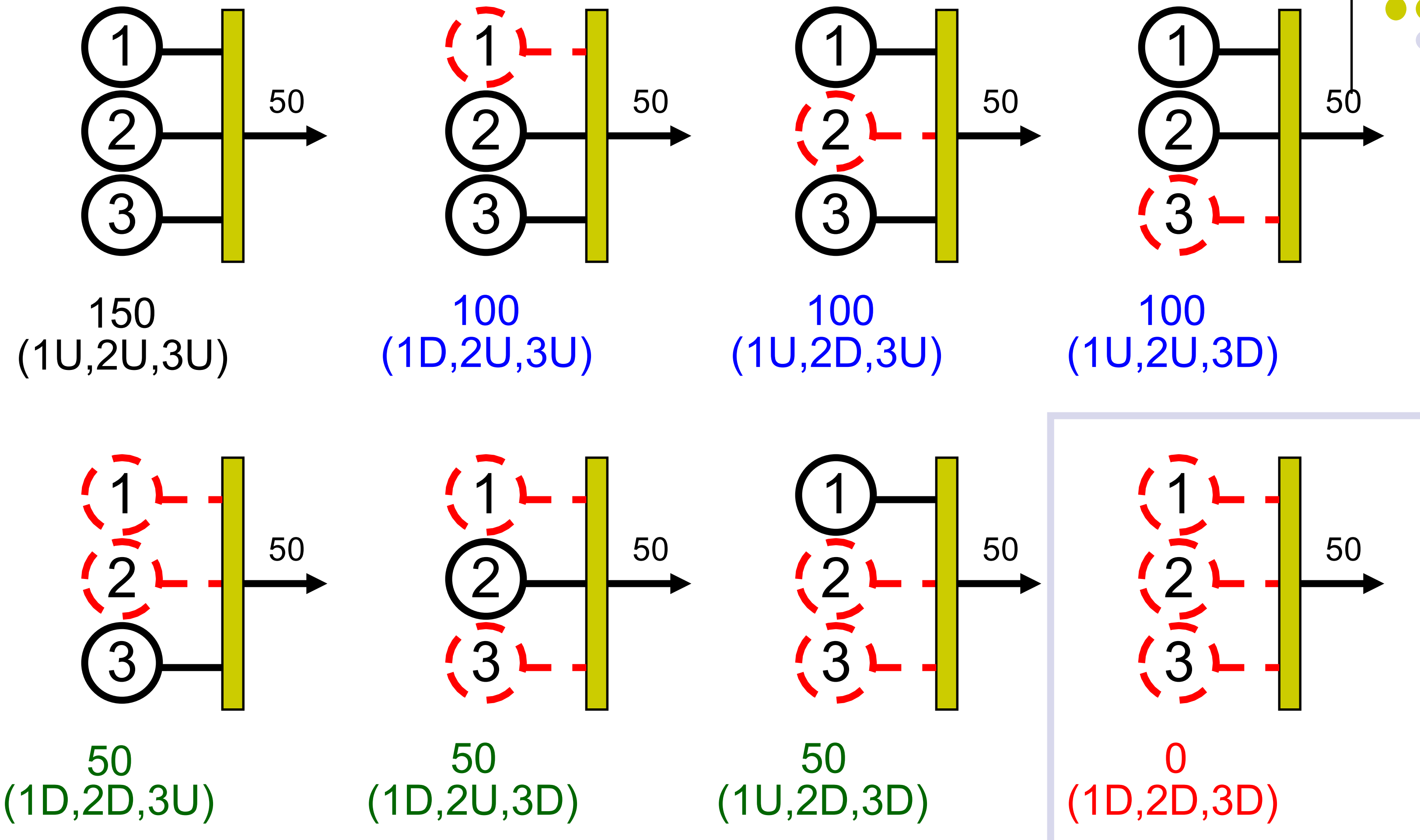
LOLP Calculation

- Let
  - B_1 be an event that load is 50 MW
  - B_2 be an event that load is 100 MW
  - B_3 be an event that load is 150 MW
- Then, using *conditional probability theory*,

$$P(E) = P(E|B_1)\times P(B_1) + P(E|B_2)\times P(B_2) + P(E|B_3)\times P(B_3)$$

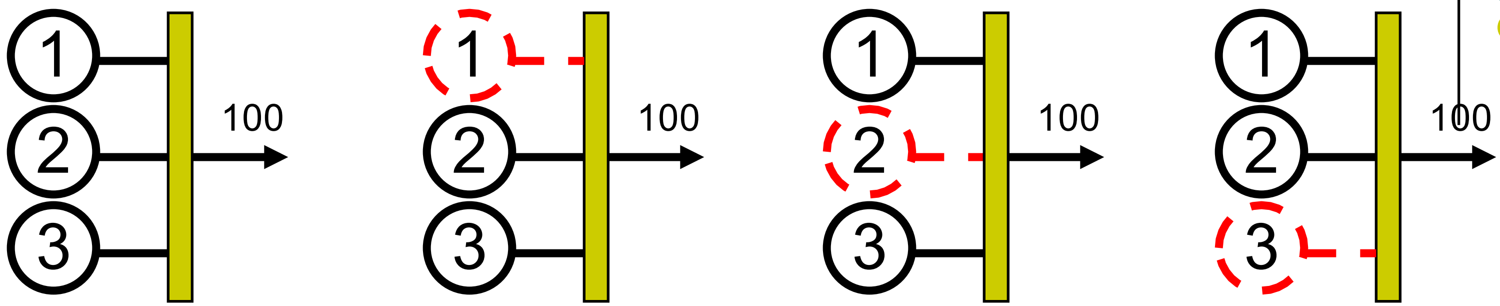
$$P(E) = P(E|B_1)\times 0.20 + P(E|B_2)\times 0.75 + P(E|B_3)\times 0.05$$

Load 50 MW

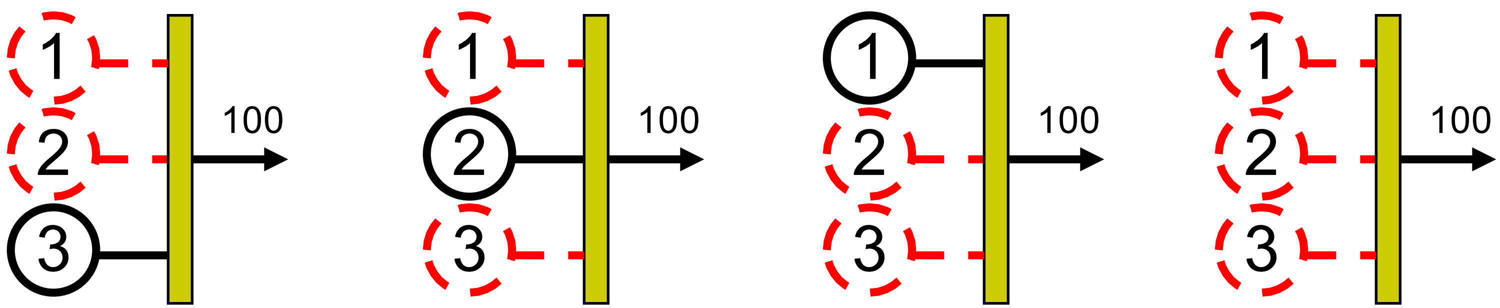


$$P(E|B_1) = P\{\ (1D,2D,3D)\ \} = 0.000001$$

Load 100 MW



150 (1U,2U,3U) | 100 (1D,2U,3U) | 100 (1U,2D,3U) | 100 (1U,2U,3D)

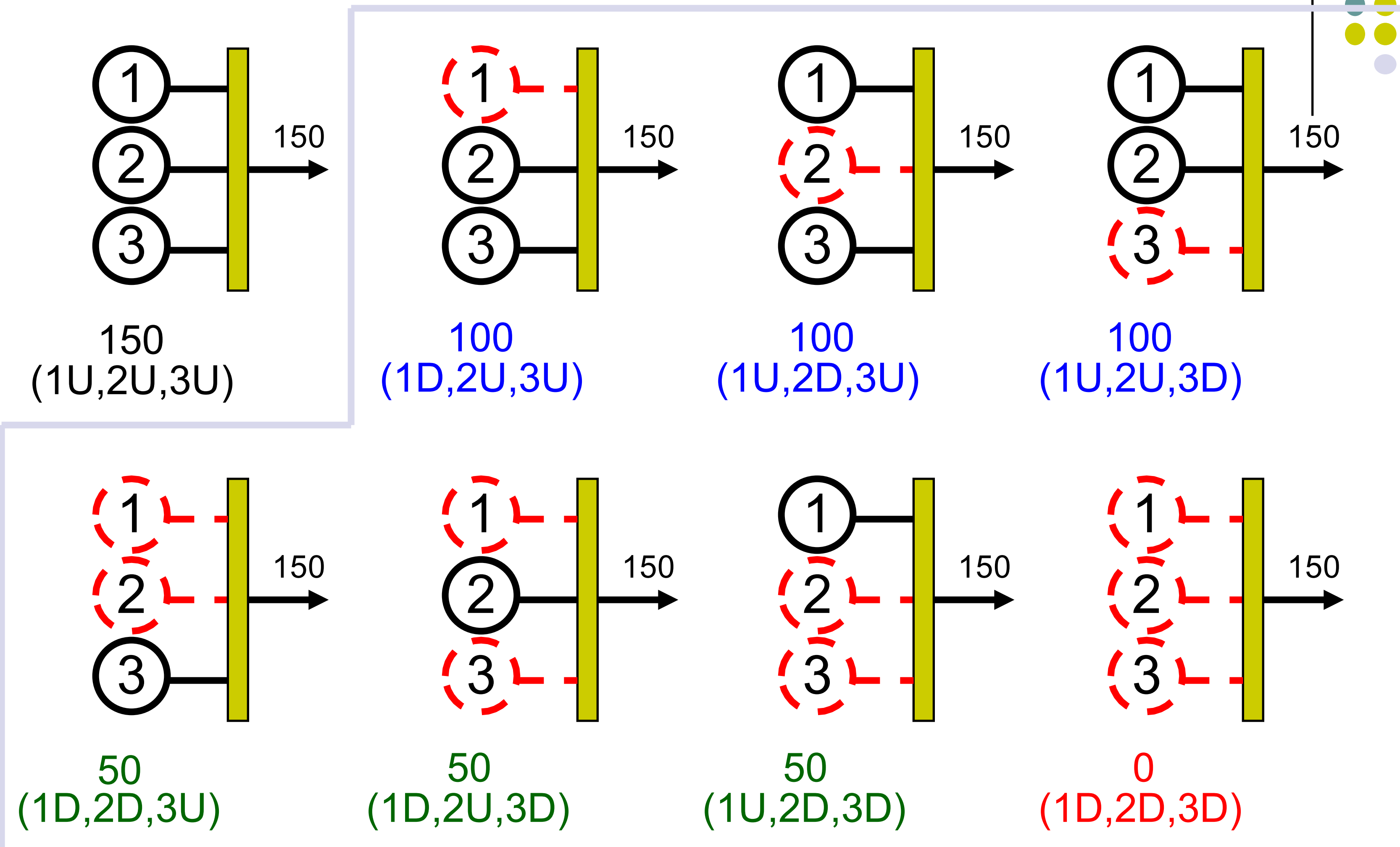


50 (1D,2D,3U) | 50 (1D,2U,3D) | 50 (1U,2D,3D) | 0 (1D,2D,3D)

$$P(E|B_2) = P\{(1D,2D,3U),(1D,2U,3D),(1U,2D,3D),(1D,2D,3D)\}$$

$$P(E|B2) = 0.000297 + 0.000001 = 0.000298$$

Load 150 MW



$$P(E|B_3) = 1 - P\{ (1U,2U,3U) \} = 0.029701$$

LOLP Calculation

- Loss of load probability is,

$$P(E) = P(E|B_1)\times 0.20 + P(E|B_2)\times 0.75 + P(E|B_3)\times 0.05$$

$$P(E) = 0.000001\times 0.20 + 0.000298\times 0.75 + 0.029701\times 0.05$$

$$P(E) = 0.00170875$$