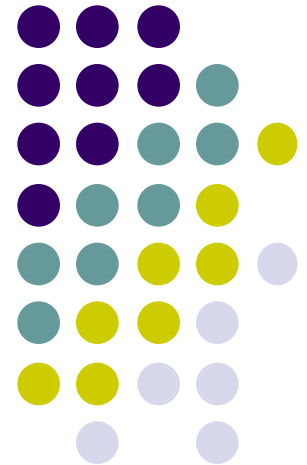


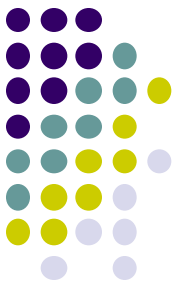
# Module 7-2

## Decomposition Approach

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# Decomposition Approach

- Now we will describe a method of decomposing the state space into subsets for the purpose of calculating the reliability indices. These sets are of the following type.
  1. Sets of acceptable states,  $A_k$  . These sets consist of states that have the load satisfied in every area.
  2. Sets of system loss of load states,  $L_k$  . These sets consist of states that have at least one area loss of load (also called unacceptable states).
  3. Sets of unclassified states,  $U_k$  . The states in these sets have not been classified into acceptable or unacceptable states.
  4. L sets are further decomposed into B sets . All states in a given B set have the same areas suffering loss of load.



# Decomposition Approach

- Now we will describe how an unclassified set can be partitioned into A, L and U subsets.
- It should be noted that in the beginning the entire state space is one U set. This U set is decomposed into A, L and U sets.
- The U sets resulting from this decomposition are further decomposed into A, L and U sets and the process goes on until a desired level of decomposition has been achieved.



# Decomposition of an Unclassified Set

- Consider an unclassified set  $S$  consisting of states  $\{x\}$  defined by a maximum state  $M$  and a minimum state  $m$  such that

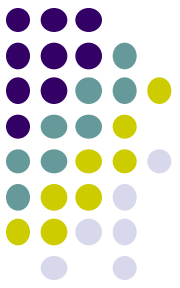
$$S = \{ x : m \leq x \leq M \}$$

where

$$M = (M_1, M_2 \dots M_n)$$

and

$$m = (m_1, m_2 \dots m_n)$$



# Decomposition of an Unclassified Set

This can be easily understood from the three area example. Here

$$M = (6 \ 7 \ 6 \ 2 \ 2 \ 2)$$

$$m = (1 \ 1 \ 1 \ 1 \ 1 \ 1)$$

The set  $S$  consists of states lying between  $M$  and  $m$ . Examples of states  $x$  between the max and min states for this example are

$$(1 \ 1 \ 1 \ 1 \ 1 \ 1)$$

$$(2 \ 1 \ 1 \ 1 \ 1 \ 1)$$

$$(2 \ 2 \ 1 \ 1 \ 1 \ 1)$$



# Decomposition of an Unclassified Set

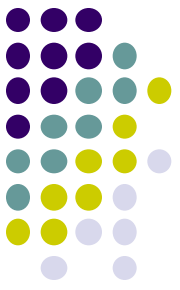
- Partition into A set:

Suppose we set all the arcs of the network model equal to the capacities in the max state and make a max flow calculation and the max flow is equal to  $D$ , the sum of area loads.

Then the max state is obviously a success state, i.e., no area suffers loss of load.

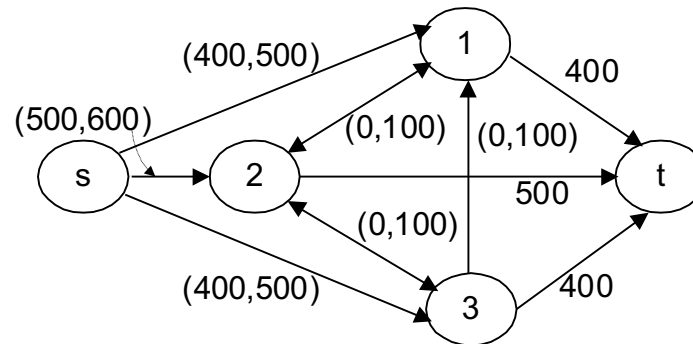
Let  $f_k(M)$  denote flow through arc  $k$  for this max flow condition. Then a vector  $u$  can be so defined that its  $k$ th element

$u_k$  = state of arc  $k$  that has capacity equal to or just greater than the flow through arc  $k$ ,  $f_k(M)$



# Decomposition of an Unclassified Set

- For the example system, the capacities corresponding to the max state and flows are shown in the diagram below:



Therefore

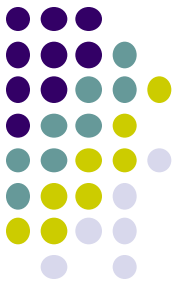
$$u = (5 \ 6 \ 5 \ 1 \ 1 \ 1)$$

Now if a state is such that capacities of all the arcs are higher than those corresponding to  $u$  then that state will also be acceptable.

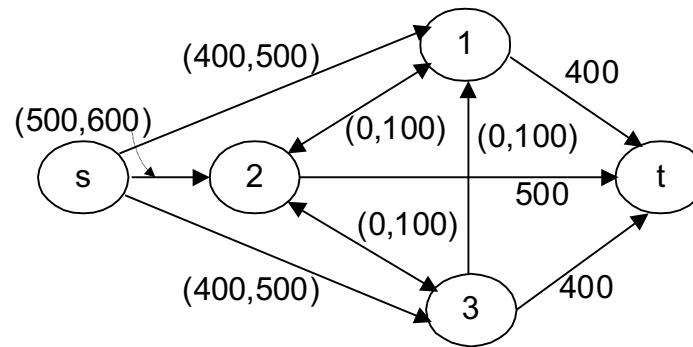
Therefore all states between the max state and  $u$  constitute an A set.

Thus

$$A = \{ x : u \leq x \leq M \}$$



# Decomposition of an Unclassified Set



For the example,

$$A = \begin{matrix} 6 & 7 & 6 & 2 & 2 & 2 \\ 5 & 6 & 5 & 1 & 1 & 1 \end{matrix}$$

The number of states in this set =  $2 \times 2 \times 2 \times 2 \times 2 = 64$







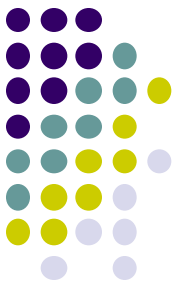
# Decomposition of an Unclassified Set

Thus we can see that just by making one flow calculation, we have been able to classify 64 states as acceptable. In the straightforward enumeration scheme we would have to make 64 flow calculations, one for each state. The probability of this set can be easily calculated as

$$P(A) = \prod_{i=1}^n \left( \sum_{m_i \leq x_i \leq M_i} P_{ix_i} \right)$$

Simply stated, first we calculate the sum of probabilities of all states between the max and min state for a given arc and then multiply these cumulated arc probabilities to find the set probability.

In actual practice the sum can be found more readily by the difference in the cumulative probabilities.



# Decomposition of an Unclassified Set

For example, for the given A set

for arc 1,

$$p_{1,5} + p_{1,6} = P_{1,6} - P_{1,4} = 1.0 - .26272 = .73728$$

for arc 2 ,

$$p_{2,6} + p_{2,7} = P_{2,7} - P_{2,5} = 1.0 - .34464 = .65536$$

for arc 3,

$$p_{3,5} + p_{3,6} = P_{3,6} - P_{3,4} = 1.0 - .26272 = .73728$$

for arc 4,

$$p_{4,1} + p_{4,2} = P_{4,2} - P_{4,0} = 1.0 - 0. = 1.0$$

for arc 5,

$$p_{5,1} + p_{5,2} = P_{5,2} - P_{5,0} = 1.0 - 0. = 1.0$$

for arc 6,

$$p_{6,1} + p_{6,2} = P_{6,2} - P_{6,0} = 1.0 - 0. = 1.0$$

Now

$$P(A1) = .73728 \times .65536 \times .73728 \times 1. \times 1. \times 1. = .35624$$

The subscript of A indicates that this is the first A set.



# Decomposition of an Unclassified Set

- Partition into L sets:

Consider arc 1 as an example. If all arcs are kept at their max capacity and the capacity of this arc is decreased, there will be a state  $v_1$  (subscript identifies the arc) for which demand in all areas is satisfied but if the system goes to the next lower state  $v_1 - 1$ , the demand is not satisfied. This identifies the first L set

$$\begin{array}{cccccccc}
 & v_1-1 & M_2 & \dots & M_{k-1} & M_k & M_{k+1} & \dots & M_n \\
 \text{max state} & & & & & & & & \\
 L_1 = & & & & & & & & \\
 \text{min state} & m_1 & m_2 & \dots & m_{k-1} & m_k & m_{k+1} & \dots & m_n
 \end{array}$$

This really means that if arc 1 ( generation in area 1 ) is between states  $v_1 - 1$  and the min state  $m_1$ , the system will have loss of load irrespective of the state of other arcs.



# Decomposition of an Unclassified Set

Similarly consider arc 2. If all arcs are kept at their max capacity and the capacity of this arc is decreased, there will be a state  $v_2$  (subscript identifies the arc) for which demand in all areas is satisfied but if the system goes to the next lower state  $v_2-1$ , the demand is not satisfied. This identifies the second L set

<b>max state</b>	$M_1$	$v_2-1$	...	$M_{k-1}$	$M_k$	$M_{k+1}$	...	$M_n$
$L_2 =$								
<b>min state</b>	$v_1$	$m_2$	...	$m_{k-1}$	$m_k$	$m_{k+1}$	...	$m_n$

Note that the arc 1 is kept between  $M_1$  and  $v_1$ . This is done to make set  $L_2$  disjoint from  $L_1$ . In other words there is no common state between  $L_1$  and  $L_2$ , because there is no common state between the states of arc 1 in these two sets.



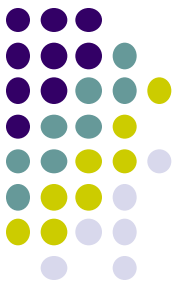
# Decomposition of an Unclassified Set

In general if there are  $n$  arcs, there will be  $n$  sets generated and the  $k$ th set will be defined by

$$\mathbf{L}_k = \begin{matrix} M_1 & M_2 & \dots & M_{k-1} & v_{k-1} & M_{k+1} & \dots & M_n \\ v_1 & v_2 & \dots & v_{k-1} & m_k & m_{k+1} & \dots & m_n \end{matrix}$$

In actual implementation to find  $v_k$ , we proceed in the following steps:

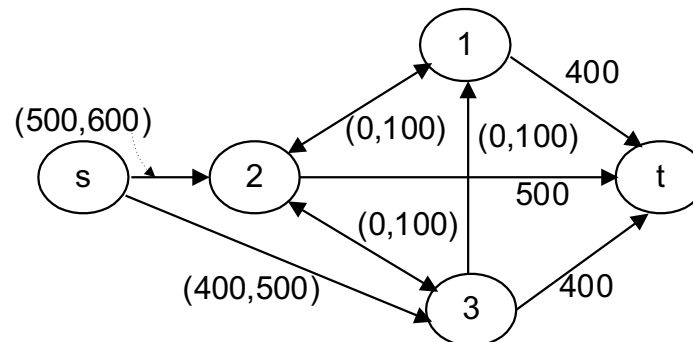
1. Set all states of the network to the maximum capacities of the U set being decomposed.
2. Find max flow using Ford- Fulkerson algorithm.
3. If the max flow found is less than the total demand, then there is loss in at least one area and thus this whole set is an L set. Otherwise go to step 4.



# Decomposition of an Unclassified Set

4. Now remove the  $k$ th arc. Keep the flows found in step 2 on all other arcs. Find the maximum flow from node  $s$  to node  $k$ . Let us say this flow is  $e_k$ . This means that even if the flow in arc  $k$  is reduced by  $e_k$ , this much flow can be sent through the unused capacity of the remaining system without having system loss of load. Thus  $v_k$  corresponds to the state with capacity equal to or just greater than  $f_k - e_k$ .

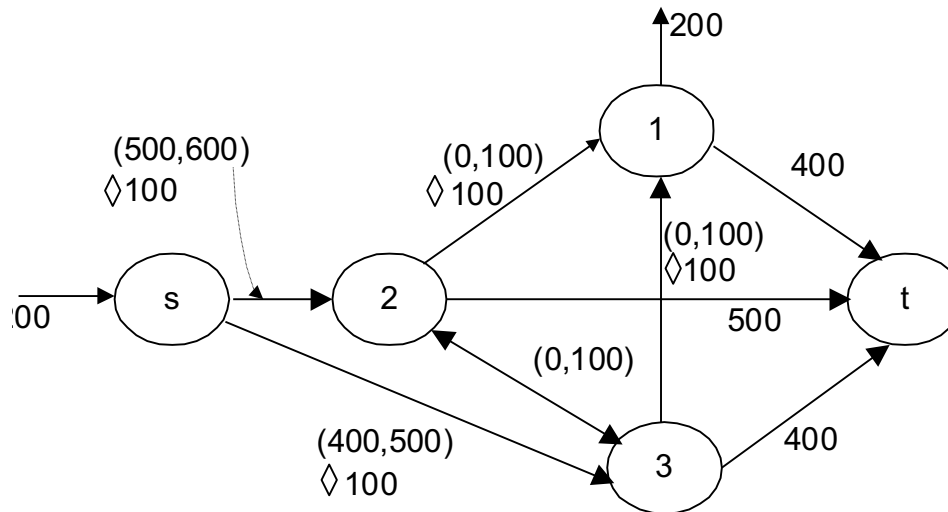
This will now be illustrated with the help of the example. The following figure shows the network with arc 1 removed and the flows found in step 2.





# Decomposition of an Unclassified Set

The following is the calculation of max flow from node s to node 1.



$$e_1 = 200$$

$$f_1 - e_1 = 400 - 200 = 200$$

The state corresponding to 200 MW in arc 1 is state 3,

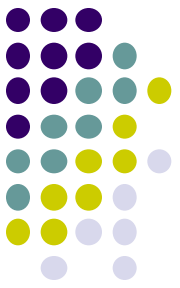
$$v_1 = 3$$

Thus

2 7 6 2 2 2

L1 =

1 1 1 1 1 1



# Decomposition of an Unclassified Set

Similarly it can be shown that

$$e_2 = 200$$

$$e_3 = 100$$

The flows on arcs 4,5,6 are 0 in step 2.

Thus

$$f_2 - e_2 = 500 - 200 = 300$$

Therefore,

$$v_2 = 4$$

$$f_3 - e_3 = 400 - 100 = 300$$

Therefore

$$v_3 = 4$$

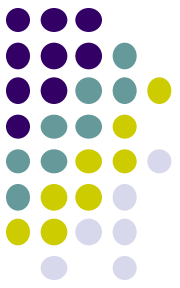
and

$$v_4 = 1$$

$$v_5 = 1$$

$$v_6 = 1$$





# Decomposition of an Unclassified Set

Therefore

$$6 \quad 3 \quad 6 \quad 2 \quad 2 \quad 2$$

L2 =

$$3 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$6 \quad 7 \quad 3 \quad 2 \quad 2 \quad 2$$

L3 =

$$3 \quad 4 \quad 1 \quad 1 \quad 1 \quad 1$$

Since the flows in arcs 4,5,6 are 0 ,there are no states in these L sets.

$$P(L1) = .00672 \times 1 \times 1 \times 1 \times 1 \times 1 = .00672$$

$$P(L2) = (1 - .00672) \times .01696 \times 1 \times 1 \times 1 \times 1 = .01685$$

$$P(L3) = (1 - .00672) \times (1 - .01696) \times .05792 \times 1 \times 1 \times 1 = 0.05656$$

$$P(L1 + L2 + L3 ) = 0.0801210844119040$$





# Decomposition of an Unclassified Set

Now some states have been classified as acceptable, some as loss of load and the remaining are as yet unclassified. One example of an unclassified set of states is

$$U_1 = \begin{array}{cccccccc} u_{1-1} & M_2 & \dots & M_{k-1} & M_k & M_{k+1} & \dots & M_n \\ v_1 & v_2 & & v_{k-1} & v_k & v_{k+1} & \dots & v_n \end{array}$$

If no arc is below its  $v$  state, then we can not say that this is necessarily an unacceptable state. If all the arcs are not at or above their  $u$  value, then we can not say that these are acceptable states. Thus it is not possible to say, at this stage, which of the states in this set are acceptable or unacceptable.

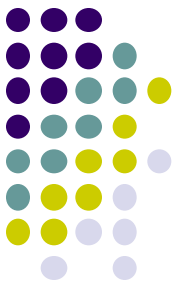


# Decomposition of an Unclassified Set

In general, there will be n number of U sets generated, the kth one defined as

$$U_k = \begin{array}{cccccccc} M_1 & M_2 & \dots & M_{k-1} & u_{k-1} & M_{k+1} & \dots & M_n \\ u_1 & u_2 & \dots & u_{k-1} & v_k & v_{k+1} & \dots & v_n \end{array}$$

The arcs preceding kth arc are kept between u and M to keep  $U_k$  disjoint from the other sets. You should remember that if at least one arc is disjoint between two sets, these sets are disjoint.



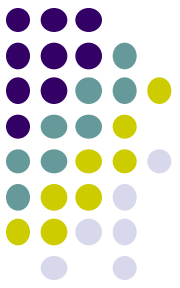
# Decomposition of an Unclassified Set

For the example,

$$U1 = \begin{matrix} 4 & 7 & 6 & 2 & 2 & 2 \\ 3 & 4 & 4 & 1 & 1 & 1 \end{matrix}$$

$$U2 = \begin{matrix} 6 & 5 & 6 & 2 & 2 & 2 \\ 5 & 4 & 4 & 1 & 1 & 1 \end{matrix}$$

$$U3 = \begin{matrix} 6 & 7 & 4 & 2 & 2 & 2 \\ 5 & 6 & 4 & 1 & 1 & 1 \end{matrix}$$



# Decomposition of an Unclassified Set

$$P(U1) = (.26272 - .00672) \times (1 - .01696) \times (1 - .05792) \times 1 \times 1 \times 1 = 0.23708$$

$$P(U2) = (1 - .26272) \times (.34464 - .01696) \times (1 - .05792) \times 1 \times 1 \times 1 = 0.22760$$

$$P(U3) = (1 - .26272) \times (1 - .34464) \times (.26272 - .05792) \times 1 \times 1 \times 1 = 0.09896$$

$$P(U1 + U2 + U3) = 0.563637148188672$$

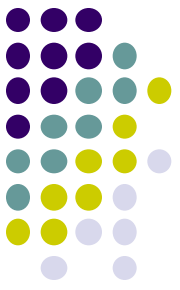
This completes the first stage of decomposition so far as system LOLE is concerned. At this stage we know that

$$LOLP \geq P(L1 + L2 + L3) = 0.0801210844119040$$



# Decomposition of an Unclassified Set

- Now if you add the probabilities of A, U and L sets they add to 1.
- Now each of the U sets is further decomposed into A, L and U sets until either no U set is left or the prob of remaining U sets is less than a threshold value.
- A schematic of the decomposition process is shown on the next page.
  
- Now system loss of load probability and EUE can be calculated from L sets. However to calculate area indices, the L sets need to be further decomposed into B sets having identical area load loss characteristics. Partitioning into B sets is a little complicated, but the basic concept can be understood as follows.



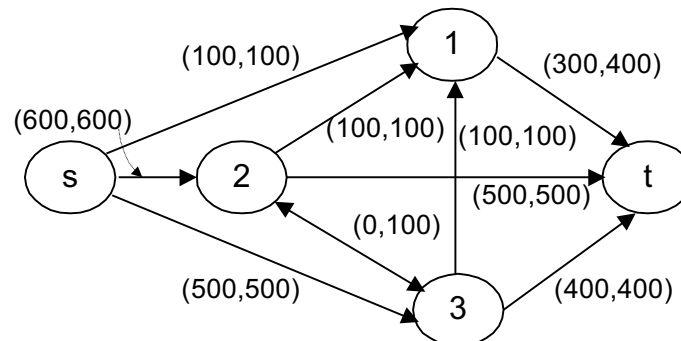
# Decomposition of an Unclassified Set

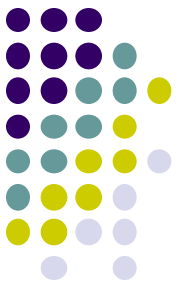
Consider set  $L_1$  having  $M^*$  and  $m^*$  as the max and min states. The capacity states of the max state are assigned to the network model and Ford-Fulkerson algorithm applied to find the max flow. By this process the max flow and flow in each arc is known. Let  $w_k$  be the state in arc  $k$  with capacity equal to or just greater than the flow in the arc. Then set  $B$  is defined as

$$B = \{x : w \leq x \leq M^*\}$$

The remaining states in the  $L$  set can be decomposed into  $n$  number of  $W$  sets as shown in the figure. These  $W$  sets are system loss of load sets and need to be decomposed into  $B$  sets.

Take the example of decomposition of  $L_1$  set. Its maximum state and flow through the arcs are shown on the arcs in the diagram below:





# Decomposition of an Unclassified Set

The B set can thus be defined as

$$B1 = \begin{matrix} 2 & 7 & 6 & 2 & 2 & 2 \\ 2 & 7 & 6 & 2 & 1 & 2 \end{matrix}$$

This set has only two states :

$$(2\ 7\ 6\ 2\ 2\ 2)$$

and

$$(2\ 7\ 6\ 2\ 1\ 2)$$



# Decomposition of an Unclassified Set



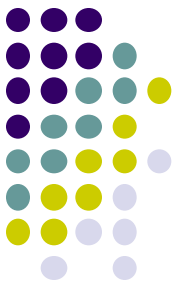
In this set all states have area 1 suffering loss of load. The W sets are :

$$W1 = \begin{matrix} 1 & 7 & 6 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$$

$$W2 = \begin{matrix} 2 & 6 & 6 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 7 & 5 & 2 & 2 & 2 \end{matrix}$$

$$W3 = \begin{matrix} 2 & 7 & 1 & 1 & 1 & 1 \end{matrix}$$

$$W4 = \begin{matrix} 2 & 7 & 6 & 1 & 2 & 2 \\ 2 & 7 & 6 & 1 & 1 & 1 \end{matrix}$$



# Decomposition of an Unclassified Set

$$W5 = \begin{matrix} 2 & 7 & 6 & 2 & 0 & 2 \\ 2 & 7 & 6 & 2 & 1 & 1 \end{matrix}$$

W5 is therefore a null set.

$$W6 = \begin{matrix} 2 & 7 & 6 & 2 & 2 & 1 \\ 2 & 7 & 6 & 2 & 1 & 1 \end{matrix}$$

A schematic of L set decomposition is shown on the next page.



# Calculation of Reliability Indices

For the given load ,

- System LOLP

$$LOLP_s = \sum_i P(L_i)$$

- LOLP for area k

$$LOLP_k = \sum_i P(B_{ki})$$

For area k

$$EUD_k = \sum_i (D_k - f_{i,k})P(B_i)$$

where  $f_{i,k}$  is the average supply to area k in  $B_k$

$$EUD = \sum_k EUD_k$$

# Stopping Criterion and The Concept of Decomposition-Simulation



- If the decomposition were to be performed exhaustively, then there would be only A and B sets. In practice, as the number of iterations of decomposition increases, the number of states classified into each new set decreases. Therefore, exhaustive decomposition becomes inefficient.
- One solution for this problem is that decomposition be stopped as the probability of non-decomposed sets drops below a certain threshold and just ignore the probability of these non-decomposed sets. This of course will introduce some approximation and is analogous to truncating capacity outage probability tables.



# Stopping Criterion and The Concept of Decomposition-Simulation



- The second solution is to estimate contribution to reliability indices from these non-decomposed sets using Monte Carlo sampling. Since this sampling is from the tail region, this can be achieved very efficiently. This method of decomposition phase followed by a simulation is called Decomposition- Simulation method.
- The contribution to the system LOLP from the simulation phase is estimated from the non-decomposed U sets. The contribution to the area LOLP and EUD indices is estimated from the non-decomposed U,L and W sets.

# Index Estimation Using Proportional Sampling



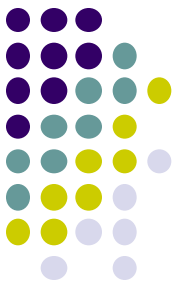
- Details of sampling will be discussed later while describing the Monte Carlo method but the basic idea can be explained now. Let us assume that we have a set of states  $S$  and that we want to estimate the probability of a subset  $A$  using sampling.
- Sampling procedure can be so arranged that each state from  $S$  is drawn proportional to its probability. Then if  $N$  states are drawn and out of these  $n_a$  satisfy the condition of being member of  $A$

$$P(A) = \frac{n_a}{N} \quad n_a \rightarrow \infty$$

# Index Estimation Using Proportional Sampling



- Based on this concept , the following procedure can be used in the simulation phase.
  1. Compute the probability of each non-decomposed set.
  2. Draw a set from all the non-decomposed sets using proportional sampling.
  3. Draw a state from the selected set using proportional sampling.
  4. Make a flow calculation and identify if it is acceptable or unacceptable. If unacceptable, identify areas suffering loss of load and the magnitude of loss of load.
  5. After a sufficient number of states have been drawn, estimate LOLP and EUD contributions from the simulation phase.



# Multi-Area Load Model

- We have based all our previous discussion on fixed loads in the areas. The load is changing continuously in different areas and there is a certain amount of correlation between the load of different areas. Let the load for hour h be represented by

$$L^h = (L_1^h, L_2^h, L_3^h, \dots, L_{n_a}^h)$$

where,

$L_j^h$  = load for hour h in area j during interval k

$n_a$  = number of areas.

- These hourly loads can be represented , using clustering algorithms, by a discrete joint probability distribution

$$d^i = (d_1^i, d_2^i, d_3^i, \dots, d_{n_a}^i) \quad i = 1, 2, n_l$$

where,

$d_j^i$  = load in area j in load state i,

$n_l$  = number of load states in the load model.



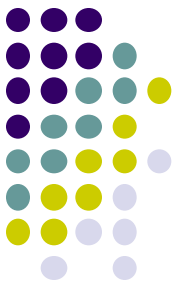


# Multi-Area Load Model

- The distribution is then represented by,

$$\Pr(D = d^i) = \text{probability of load state } d^i.$$

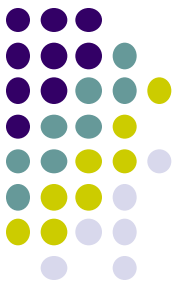
- A method to determine this discrete joint distribution is explained next.



# Indices Including the Load Model

- By using the load model described, hourly loads can be represented by 10 to 20 load states. Each load state has a probability and fixed values of load in each area. Now indices can be computed for each load state and over all indices found by aggregation

$$\begin{aligned} & \textit{Index} \\ &= \sum_i (\textit{index for load state } i) * (\textit{Prob of load state } i) \end{aligned}$$



# Basic Clustering Algorithm

- One suitable approach to deriving an interconnected system load model of the above form is to use the K-means clustering algorithm. These algorithms group hourly values of area load based on their proximity, and result in nonequispaced multi-area load states.
- Starting with hourly values of area loads, a choice of number of clusters (load states in the model), and initial cluster means, the algorithm performs a disjoint cluster analysis on the basis of Euclidean distances from each hourly observation to the different cluster means. This process is called nearest centroid sorting, and can be described in the following way. Let

NHS = number of hours in the period of study

$L_{kj}$  = load demand in area  $j$  during  $k$ th hour

NC = number of clusters or load states in the model



# Basic Clustering Algorithm

- The various steps are described below.
  1. Choose initial seeds, or initial values of cluster means and assign them to  $MEAN_{ij}$   
 $MEAN_{ij}$  = mean value of load demand in area j for cluster i
  2. Obtain an initial partition of the hourly load demands into clusters. This is achieved by computing Euclidean distances from each hourly load data unit to each cluster, and assigning them to the nearest cluster. Euclidean distances from each data unit to each cluster mean are computed by

$$DIST_{ki} = \sqrt{\sum_{j=1}^N (MEAN_{ij} - L_j^k)^2}$$

where

$DIST_{ki}$  = Euclidean distance from the kth data unit of ith cluster mean



# Basic Clustering Algorithm

3. Compute new cluster means by computing the mean of the data units belonging to each cluster

$$MEAN_{ij} = \sum_{k \in \text{ith cluster}} \frac{L_j^k}{NM_i}$$

where

$NM_i$  = number of data units belonging to  $i$ th cluster

4. Take each data unit in sequence and compute Euclidean distances to all clusters. If nearest cluster is not the cluster to which the data unit belongs, reassign the data unit and update the cluster means of the gaining and losing clusters.
5. Repeat step 4 until convergence is achieved, that is, continue until a full cycle through data units fails to cause any change in cluster membership.



# Basic Clustering Algorithm

6. After convergence is achieved, values of load for each load state are given by cluster means, in other words,

$$d_j^i = MEAN_{ij}, \text{ for } i = j, \dots, n_a$$

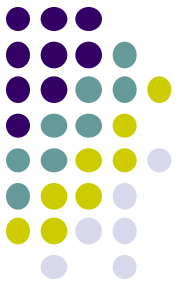
and probability and frequencies are computed by

$$P(D = d^j) = \frac{NM_j}{NHS}$$

$$F(d^i - d^l) = NLC_{il}$$

where

$NLC_{il}$  = number of times a change in hourly load caused a change in cluster membership from cluster  $i$  to cluster  $l$ .



# Example of two area clustering

This example illustrates how seven observations are grouped into two clusters. The following is the data on observations.

Obs	$X_1$	$X_2$
1	3	2
2	2	6
3	6	7
4	5	4
5	5	8
6	3	4
7	4	6



# Example of two area clustering

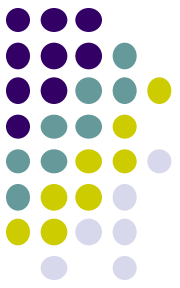
Solution:

## Pass 1

Cluster	Seeds	Distances to cluster seeds				
		Obs	Coord	1	2	
1	(3,2)	3	(6,7)	$\sqrt{34}$	$\sqrt{17}$ X	
2	(2,6)	4	(5,4)	$\sqrt{8}$ X	$\sqrt{13}$	
		5	(5,8)	$\sqrt{40}$	$\sqrt{13}$ X	
		6	(3,4)	$\sqrt{4}$ X	$\sqrt{5}$	
		7	(4,6)	$\sqrt{17}$	$\sqrt{4}$ X	

The mark, X, denotes smaller figure between the distances to cluster seeds.





# Example of two area clustering

Then, new centroids are computed:

From (3,2), (5,4), (3,4),  $\hat{x}_1 = 3.67, \hat{x}_2 = 3.33$

From (2,6), (6,7), (5,8), (4,6),  $\hat{x}_1 = 4.75, \hat{x}_2 = 6.75$

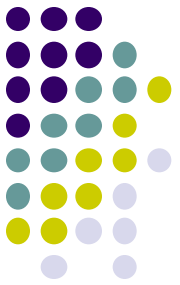
Now seeds are then

(3.67,3.33)      (4.25,6.75)

Thus, distances between old and new seeds

$$D_1 = \sqrt{(3.67 - 3)^2 + (3.33 - 2)^2} = 1.49$$

$$D_2 = \sqrt{(4.25 - 2)^2 + (6.75 - 6)^2} = 2.3717$$



# Example of two area clustering

Pass 2

Distances to cluster seeds

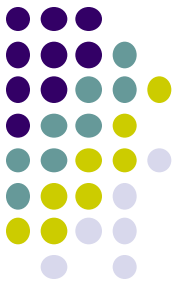
Cluster	Seeds	Obs	Coord	1	2
1	(3.67,3.33)	1	(3,2)	1.489	4.911
		2	(2,6)	3.149	2.371
		3	(6,7)	4.347	1.767
2	(4.25,6.75)	4	(5,4)	1.489	2.850
		5	(5,8)	4.855	1.458
		6	(3,4)	0.947	3.020
		7	(4,6)	2.690	0.790

Then, new centroids are computed:

From (3,2), (5,4), (3,4),  $\hat{x}_1 = 3.67, \hat{x}_2 = 3.33$

From (2,6), (6,7), (5,8), (4,6),  $\hat{x}_1 = 4.75, \hat{x}_2 = 6.75$





# Example of two area clustering

Now seeds are then

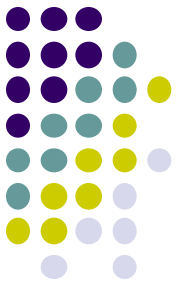
$$(3.67, 3.33) \quad (4.25, 6.75)$$

Thus, distances between old and new seeds

$$D_1 = \sqrt{(3.67 - 3.67)^2 + (3.33 - 3.33)^2} = 0$$

$$D_2 = \sqrt{(4.25 - 4.25)^2 + (6.75 - 6.75)^2} = 0$$

Since convergence criteria are met, clustering is done. This procedure is shown graphically in next page



# Example of two area clustering

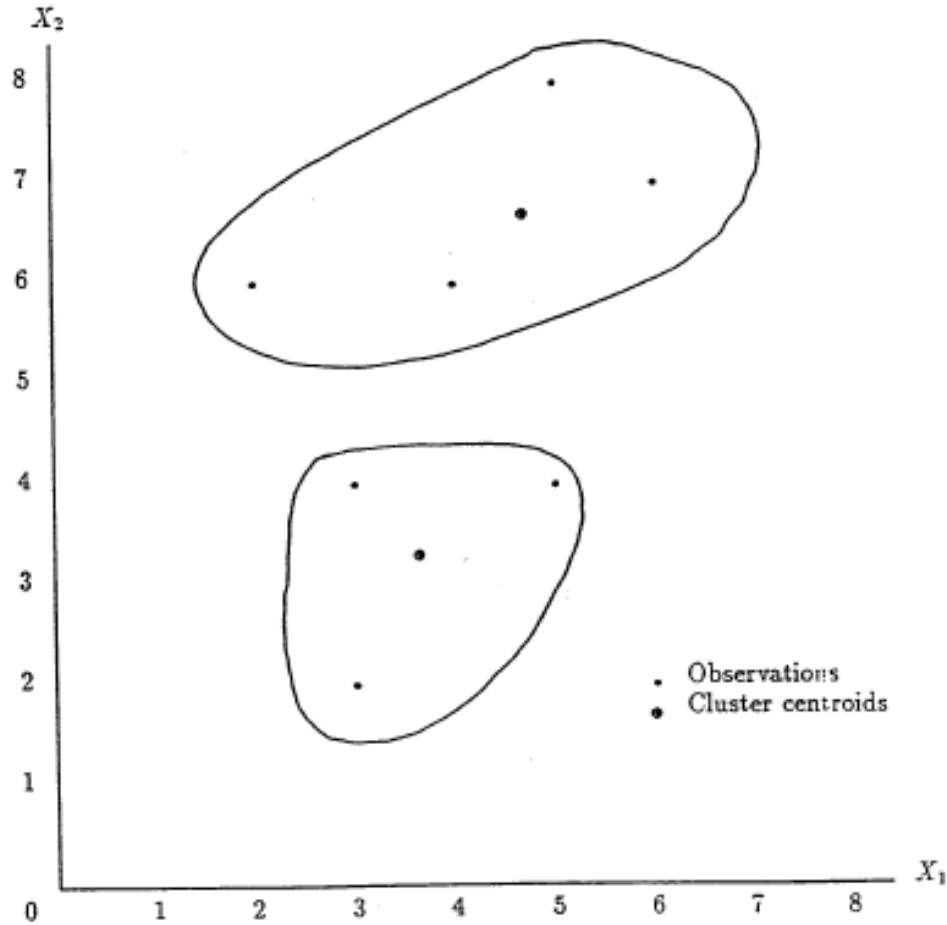


Figure Clustering by FASTCLUS procedure