ECEN643
Problem Solving Session_2

Content

There will be 2 parts.

1st part is review of previous lectures on decomposition

2nd part is solving a example problem using two approaches

Review of Decomposition

1. Ford-Fulkersen Algorithm

2. Understanding of u vector

3. Understanding of v_k

4. Understanding of A, L, U sets

F-F Algorithm

1. Start with a feasible flow on all arcs. Capacity restrictions and flow conservation at nodes must be satisfied.
2. Using labeling routine, find a flow augmenting path s to n through which a positive flow can be sent.
3. Compute the maximal flow δ that can be sent along the path.
4. Increase the flow on all forward arcs on the path by δ .
5. Find another flow augmenting path and repeat the procedure.

The algorithm terminates when no flow augmenting path can be found.

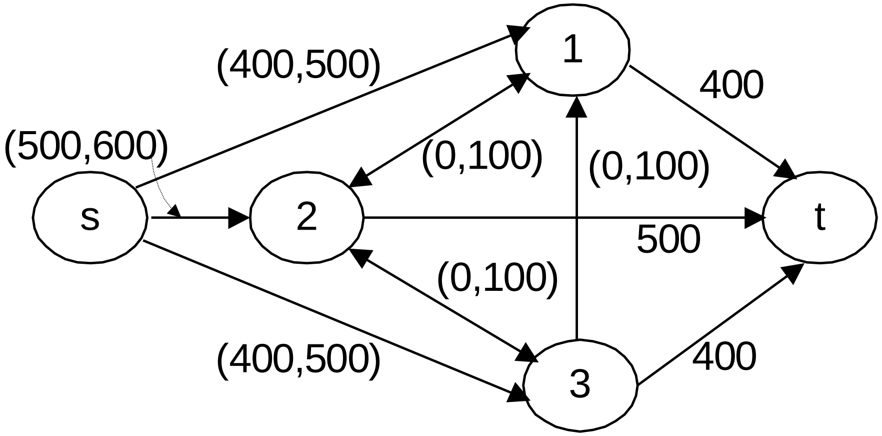
Designed for large and complicated system

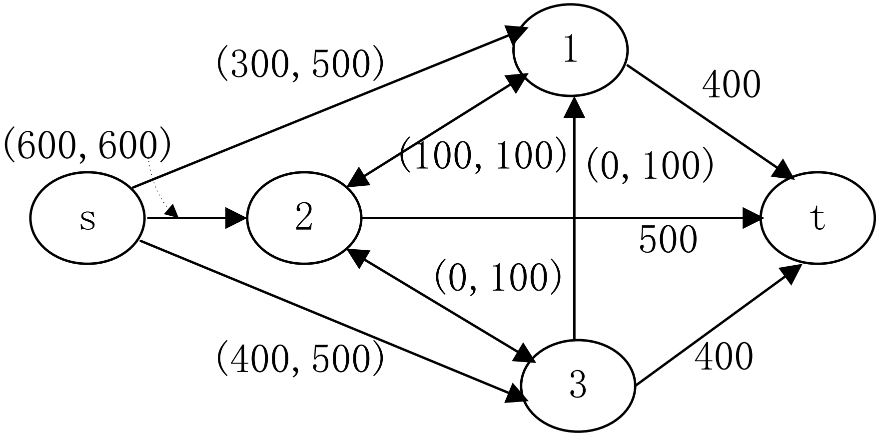
F-F Algorithm

1. Try to supply the load with minimum generation possible

2. Multiple solutions possible

3. To reduce the computation complicity, try to make more arcs with 0 flow. So the failure of that arc will not affect the system, thus less L sets and U sets





Understanding of u vector

U vector directly corresponds to results of F-F algorithm

If every arc is above or equal to u vector, system in acceptable sets (A)

If every arc is below u vector, system in loss of load sets. (L)

If some arc is above and some is below, system is in unclassified sets (U)

Understanding of v_k

v_k for arc k
Individual definition, different from u vector

If arc k is below v_k

even if other arcs are trying their best to supply the load

system is in loss of load sets. (L_k)

v_k: Minimum state needed for arc k to supply the load

Understanding of v_k

Steps to find v_k

In actual implementation to find v_k, we proceed in the following steps:

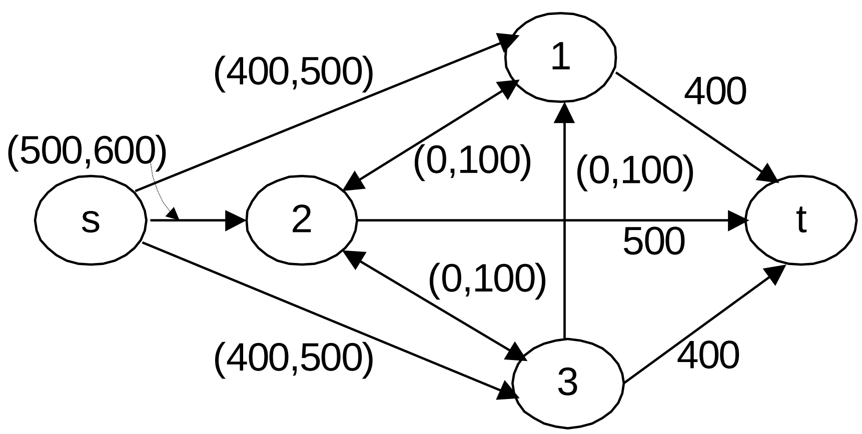
1. Set all states of the network to the maximum capacities of the U set being decomposed.
2. Find max flow using Ford- Fulkerson algorithm.
3. If the max flow found is less than the total demand, then there is loss in at least one area and thus this whole set is an L set. Otherwise go to step 4.
4. Now remove the kth arc. Keep the flows found in step 2 on all other arcs. Find the maximum flow from node s to node k. Let us say this flow is e_k . This means that even if the flow in arc k is reduced by e_k, this much flow can be sent through the unused capacity of the remaining system without having system loss of load. Thus v_k corresponds to the state with capacity equal to or just greater than $f_k - e_k$.

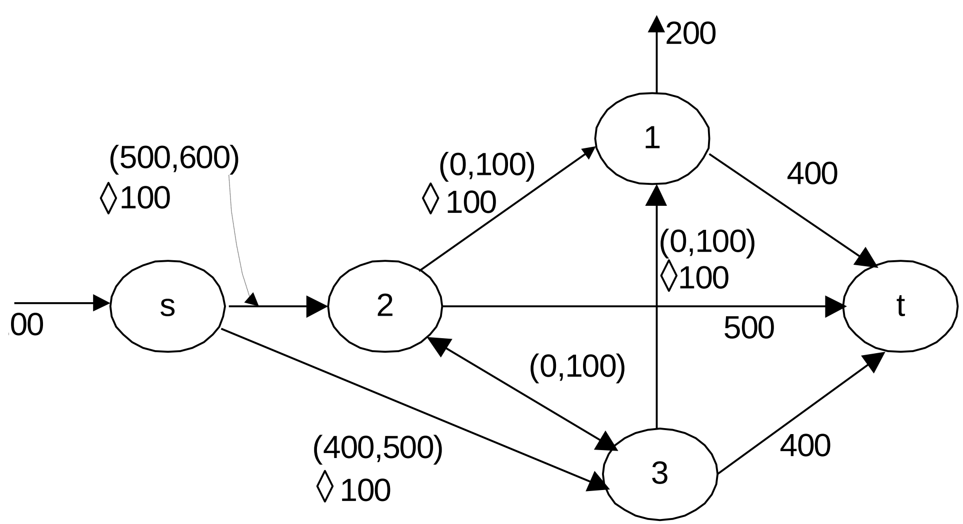
Understanding of v_k

f_k: original flow in F-F algorithm, flow needed to supply the load

e_k: Maximum flow can be supplied between two nodes of arcs k, if arc k is removed

$f_k - e_k$: minimum flow needed to supply the load, with all other arcs trying their best





Understanding of A L U Sets

If every arc is above or equal to u vector, system in acceptable sets (A)

$$A = \begin{pmatrix} M_1 & M_2 & \dots\dots & M_n \\ u_1 & u_2 & \dots\dots & u_n \end{pmatrix}$$

Understanding of A L U Sets

If arc k is below v_k

even if other arcs are trying their best to supply the load

system is in loss of load sets. (L_k)

$$L_1 = \begin{pmatrix} v_1 - 1 & M_2 & \dots \dots & M_n \\ m_1 & m_2 & \dots \dots & m_n \end{pmatrix}$$

$$L_2 = \begin{pmatrix} M_1 & v_2 - 1 & \dots \dots & M_n \\ v_1 & m_2 & \dots \dots & m_n \end{pmatrix}$$

$$L_k = \begin{pmatrix} M_1 & \dots \dots & v_k - 1 & \dots \dots & M_n \\ v_1 & \dots \dots & m_k & \dots \dots & m_n \end{pmatrix}$$

Understanding of A L U Sets

U sets are remaining sets of state space S after A and L sets are determined

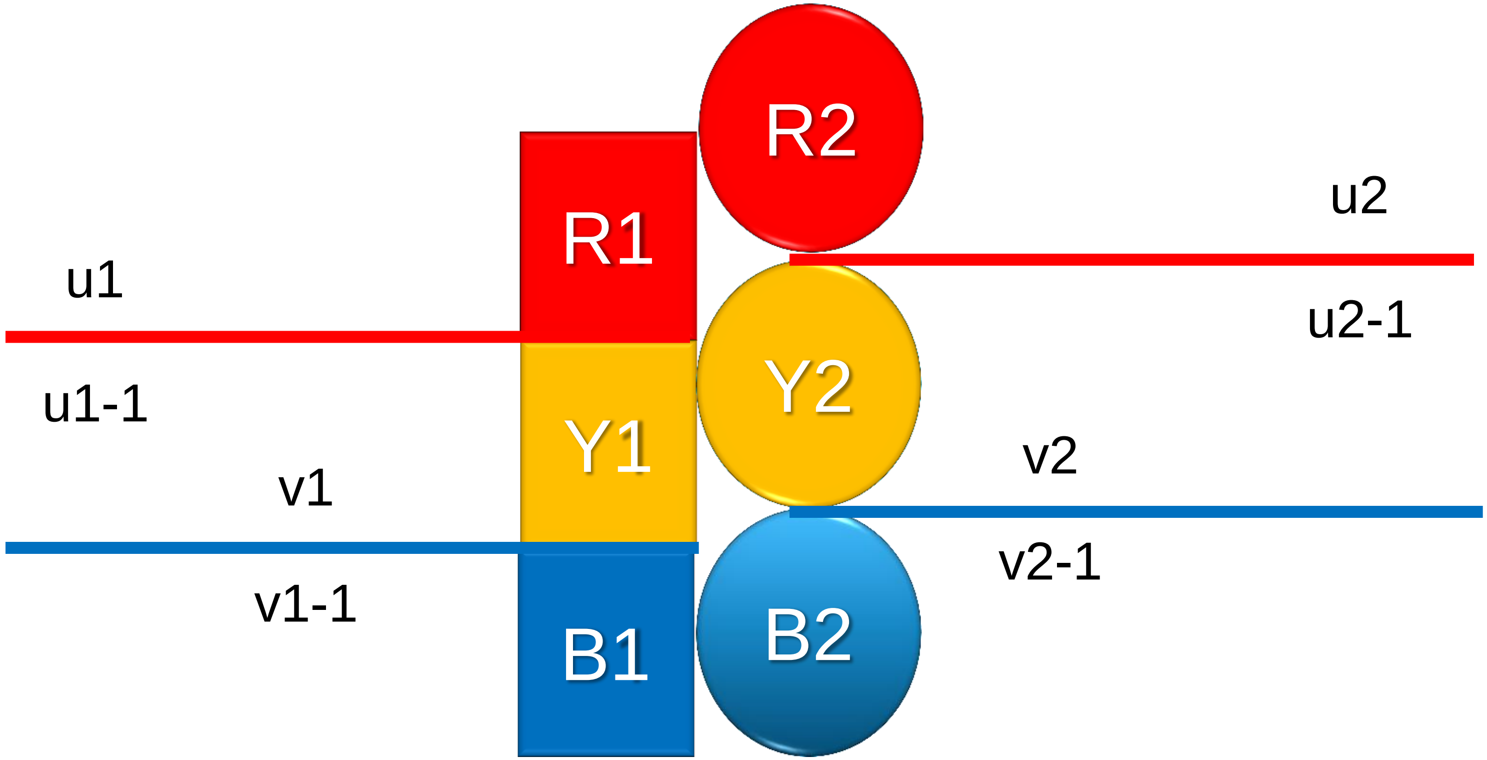
$$U_1 = \begin{pmatrix} u_1 - 1 & M_2 \; \dots \dots & M_n \\ v_1 & v_2 \dots \dots & v_n \end{pmatrix}$$

$$U_2 = \begin{pmatrix} M_1 & u_2 - 1 \dots \dots & M_n \\ u_1 & v_2 \dots \dots & v_n \end{pmatrix}$$

$$U_k = \begin{pmatrix} M_1 \; \dots \dots & u_k - 1 \dots \dots & M_n \\ u_1 \; \dots \dots & v_k \; \dots \dots & v_n \end{pmatrix}$$

Understanding of A, L, U sets

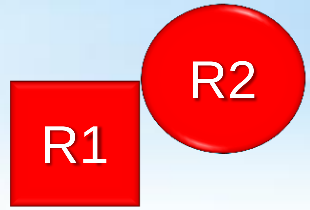
A simple system with two arcs



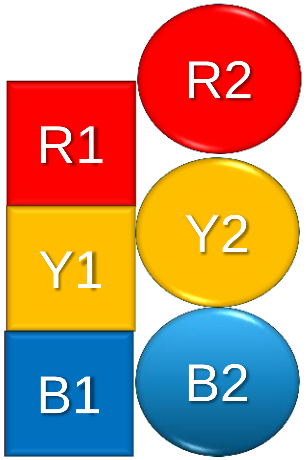
9 possible combination

1

A

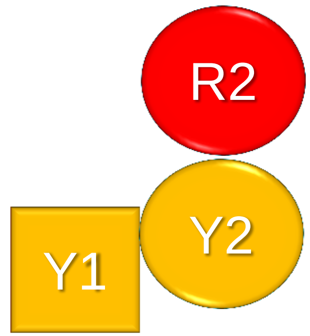


$$A = \begin{pmatrix} M_1 & M_2 & \dots\dots & M_n \\ u_1 & u_2 & \dots\dots & u_n \end{pmatrix}$$

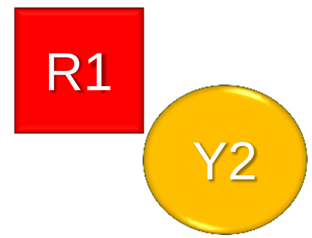


2+1

U1



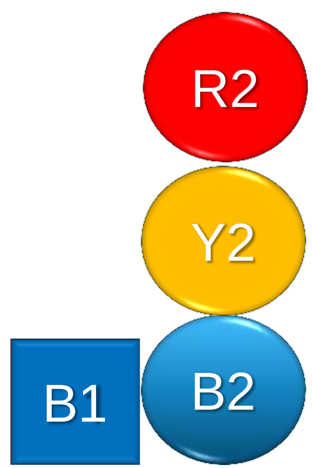
$$U_k = \begin{pmatrix} M_1 & \dots\dots & u_k - 1 & \dots\dots & M_n \\ u_1 & \dots\dots & v_k & \dots\dots & v_n \end{pmatrix}$$



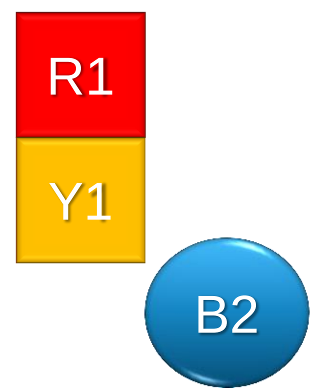
U2

3+2

L1



$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 & \dots\dots & M_n \\ v_1 & \dots\dots & m_k & \dots\dots & m_n \end{pmatrix}$$

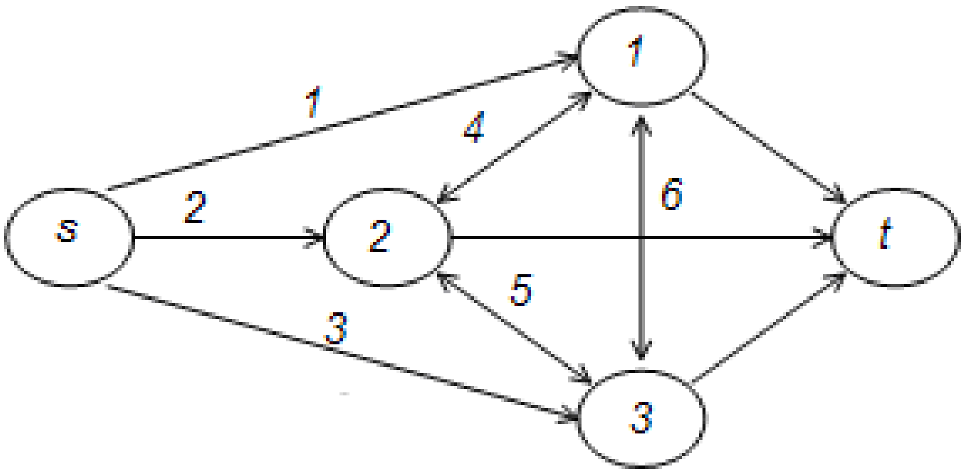


L2

=9

Example Problem

Assume a system of 3 interconnected (Arc number on respective arrows):

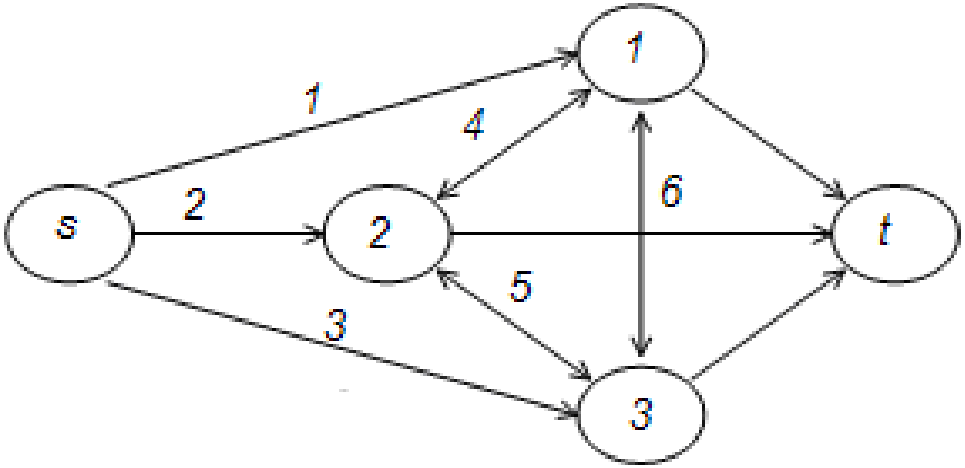


<u>Generation:</u> There are 4 identical generators in each area, each with a capacity of 200 MW and probability of failure 0.15

<u>Transmission:</u> The transmission between any two areas has a bidirectional capacity of 200 MW and is assumed not to fail.

<u>Load:</u> The load in each area is assumed to be 700 MW.

Example Problem



<u>Question 1:</u> Please find the A, L and U sets resulting from the first decomposition.

<u>Question 2:</u> Then decompose the resulting U sets into A, L and U sets

<u>Question 3:</u> Find the probabilities of A, L and U sets from these two levels of decomposition and tabulate the trend as a function of the level of decomposition.

Example Problem

Since probability calculation is required, generation capacity outage table is needed.
To build a capacity outage table of generation system, cumulative probability is calculated from exact probability of each state.

<u>**Generation:**</u> There are 4 identical generators in each area, each with a capacity of 200 MW and probability of failure 0.15

$$p_f = 0.15, p_u = 0.85$$

$$P_1 = P(Capout \geq 0) = 1$$

$$P_2 = P(Capout \geq 200) = P_1 - P(Capout = 0) = P_1 - \binom{4}{0} p_u^4 = 0.4780$$

$$P_3 = P(Capout \geq 400) = P_2 - P(Capout = 200) = P_2 - \binom{4}{1} p_u^3 p_f = 0.1095$$

$$P_4 = P(Capout \geq 600) = P_3 - P(Capout = 400) = P_3 - \binom{4}{2} p_u^2 p_f^2 = 0.0120$$

$$P_5 = P(Capout \geq 800) = P_4 - P(Capout = 600) = P_4 - \binom{4}{3} p_u p_f^3 = 0.0005$$

Example Problem

<u>Generation:</u> There are 4 identical generators in each area, each with a capacity of 200 MW and probability of failure 0.15

Generation Capacity Outage Table

i	C_i (Capacity out)	P_i (Cum. Prob.)	Arc State
1	0 MW	1	5
2	200 MW	0.4780	4
3	400 MW	0.1095	3
4	600 MW	0.0120	2
5	800MW	0.0005	1

Example Problem

Next step is to develop arc state table for all the arcs in system

Trick:
Because transmission line won't fail, only one state for transmission arc

Arc State	Arc Capacity / MW					
	1	2	3	4	5	6
5	800	800	800			
4	600	600	600			
3	400	400	400			
2	200	200	200			
1	0	0	0	200	200	200

Example Problem

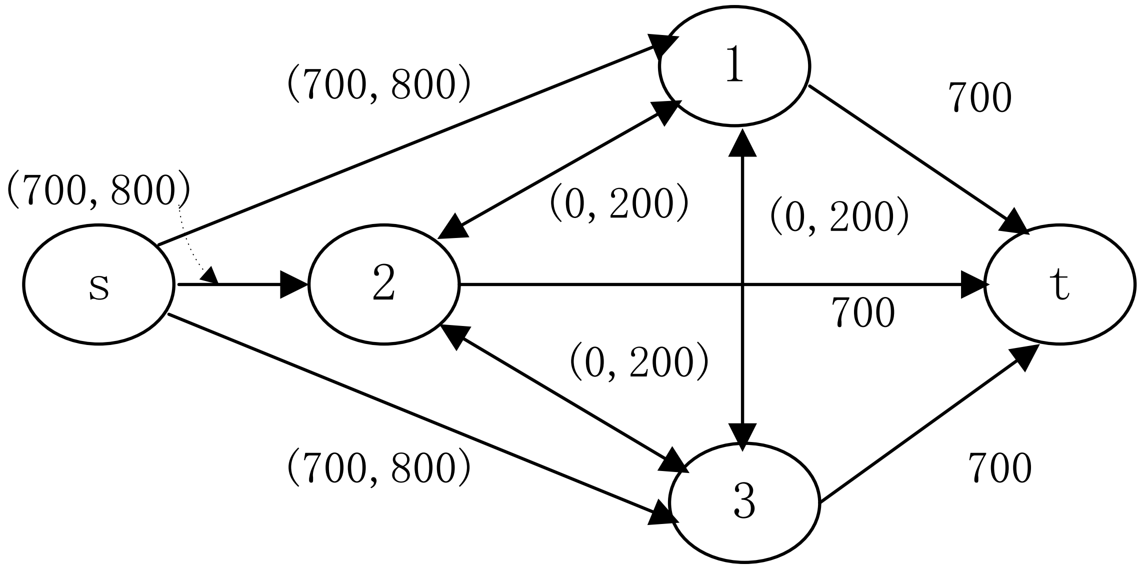
So at the beginning of first level decomposition, we have

$$S = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Arc State	Arc Capacity / MW					
	1	2	3	4	5	6
5	800	800	800			
4	600	600	600			
3	400	400	400			
2	200	200	200			
1	0	0	0	200	200	200

Example Problem

Firstly, find the maximum flow and corresponding u vector



$$u = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$$

700 flow needs 800 capacity, thus u1= u2= u3= state 5

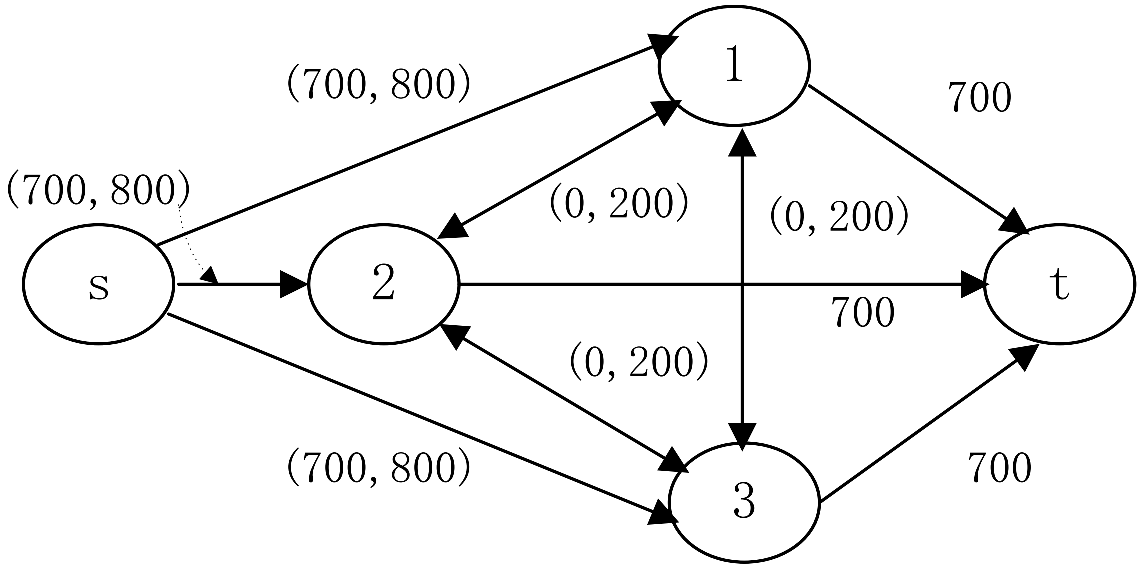
Thus, $A = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 5 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$

$$A = \begin{pmatrix} M_1 & M_2 & \dots\dots & M_n \\ u_1 & u_2 & \dots\dots & u_n \end{pmatrix}$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 & \dots\dots & M_n \\ v_1 & \dots\dots & m_k & \dots\dots & m_n \end{pmatrix}$$

Find v1 and corresponding L1



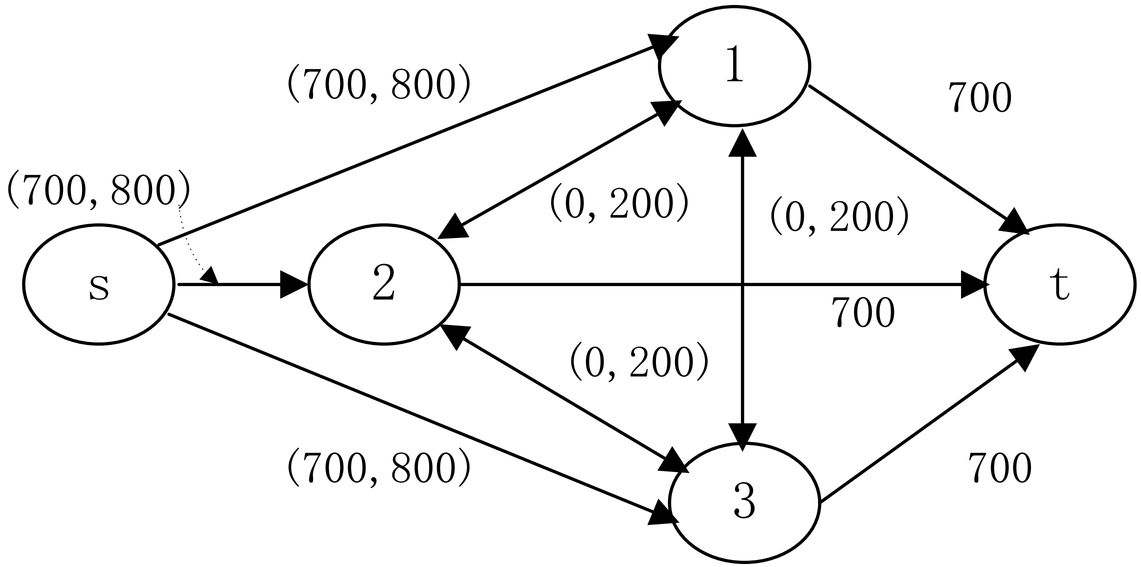
$$f1 - e1 = 700 - 200 = 500$$
$$v1 = 4\ (600\ capacity)$$

$$\text{L1} = \begin{pmatrix} 3 & 5 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 \dots\dots & M_n \\ v_1 & \dots\dots & m_k \;\; \dots\dots & m_n \end{pmatrix}$$

Find v2 and corresponding L2



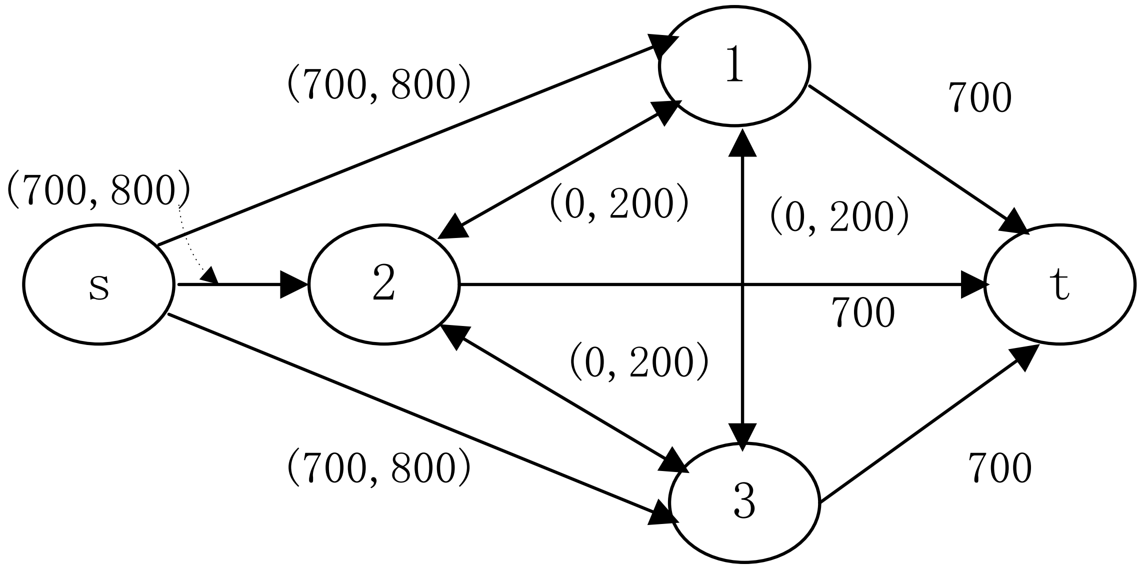
$$f2 - e2 = 700 - 200 = 500$$
$$v2 = 4\ (600\ capacity)$$

$$\text{L2} = \begin{pmatrix} 5 & 3 & 5 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 \dots\dots & M_n \\ v_1 & \dots\dots & m_k \quad \dots\dots & m_n \end{pmatrix}$$

Find v3 and corresponding L3

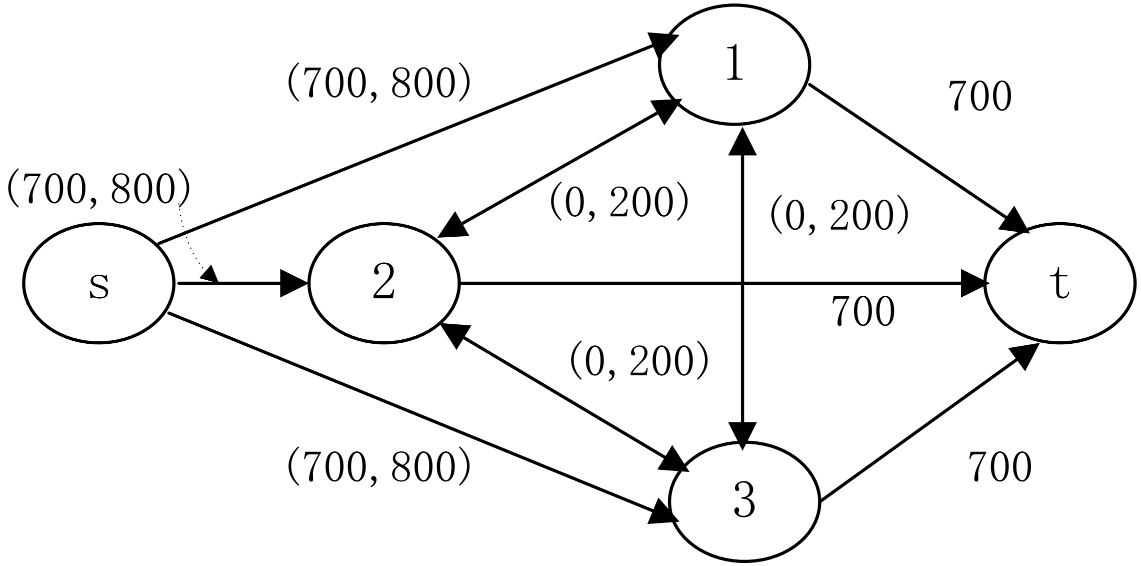


$$f3 - e3 = 700 - 200 = 500$$
$$v3 = 4\ (600\ capacity)$$

$$\text{L3} = \begin{pmatrix} 5 & 5 & 3 & 1 & 1 & 1 \\ 4 & 4 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Example Problem

Flow in arc 4,5,6 are zero, their failure would never affect the system
Besides, there is only one possible state in arc 4,5,6



$$v4 = v5 = v6 = 1$$

$$\mathrm{L4} = \mathrm{L5} = \mathrm{L6} = \emptyset$$

Example Problem

$$U_k = \begin{pmatrix} M_1 & \dots\dots & u_k - 1 \dots\dots & M_n \\ u_1 & \dots\dots & v_k \ \dots\dots & v_n \end{pmatrix}$$

Find remaining U sets

$$S = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathrm{u} = (5 \quad 5 \quad 5 \quad 1 \quad 1 \quad 1)$$
$$\mathrm{v} = (4 \quad 4 \quad 4 \quad 1 \quad 1 \quad 1)$$

$$U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$U2 = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 5 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$U3 = \begin{pmatrix} 5 & 5 & 4 & 1 & 1 & 1 \\ 5 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$U4 = U5 = U6 = \emptyset$$

End of 1st level of decomposition

i	C_i (Capacity out)	P_i (Cum. Prob.)	Arc State
1	0 MW	1	5
2	200 MW	0.4780	4
3	400 MW	0.1095	3
4	600 MW	0.0120	2
5	800MW	0.0005	1

$$A = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 5 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(A) = (P_1 - P_2) * (P_1 - P_2) * (P_1 - P_2) * 1 = 0.1422$$

$$L1 = \begin{pmatrix} 3 & 5 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, L2 = \begin{pmatrix} 5 & 3 & 5 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, L3 = \begin{pmatrix} 5 & 5 & 3 & 1 & 1 & 1 \\ 4 & 4 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L1) = (P_3) * (P_1) * (P_1) * 1 = 0.1095$$

$$Prob(L2) = (P_1 - P_3) * (P_3) * (P_1) * 1 = 0.0975$$

$$Prob(L3) = (P_1 - P_3) * (P_1 - P_3) * (P_3) * 1 = 0.0868$$

$$Prob(L) = Prob(L1) + Prob(L2) + Prob(L3) = 0.2939$$

$$U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}, U2 = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 5 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}, U3 = \begin{pmatrix} 5 & 5 & 4 & 1 & 1 & 1 \\ 5 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(U1) = (P_2 - P_3) * (P_1 - P_3) * (P_1 - P_3) * 1 = 0.2922$$

$$Prob(U2) = (P_1 - P_2) * (P_2 - P_3) * (P_1 - P_3) * 1 = 0.1713$$

$$Prob(U3) = (P_1 - P_2) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.1004$$

$$Prob(U) = Prob(U1) + Prob(U2) + Prob(U3) = 0.5639$$

$$\sum Prob = 1 = Prob(S)$$

Example Problem

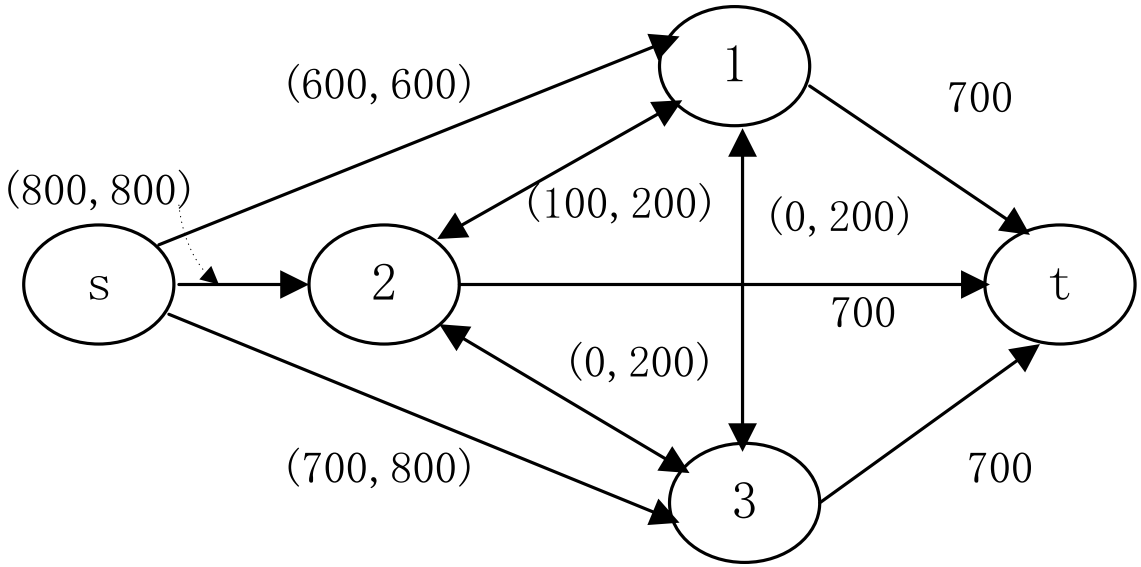
At 2nd level of decomposition, start with U1

$$S - \text{new} = \text{U1} = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

Arc State	Arc Capacity / MW					
	1	2	3	4	5	6
5	800	800	800			
4	600	600	600			
3	400	400	400			
2	200	200	200			
1	0	0	0	200	200	200

Example Problem

Firstly, find the maximum flow and corresponding u vector



$$u = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$$

700 flow needs 800 capacity, thus u3= state 5

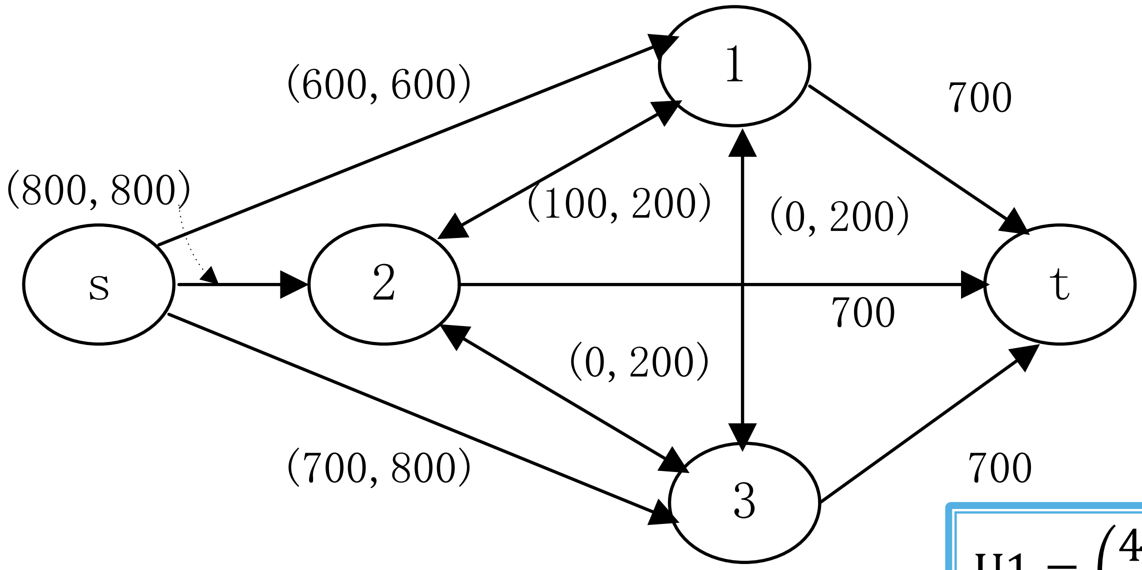
Thus, $A - U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$

$$A = \begin{pmatrix} M_1 & M_2 & \dots\dots & M_n \\ u_1 & u_2 & \dots\dots & u_n \end{pmatrix}$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 \dots\dots & M_n \\ v_1 & \dots\dots & m_k \ \dots\dots & m_n \end{pmatrix}$$

Find v1 and corresponding L11



$$U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$f1 - e1 = 600 - 100 = 500$$
$$v1 = 4\ (600\ capacity)$$

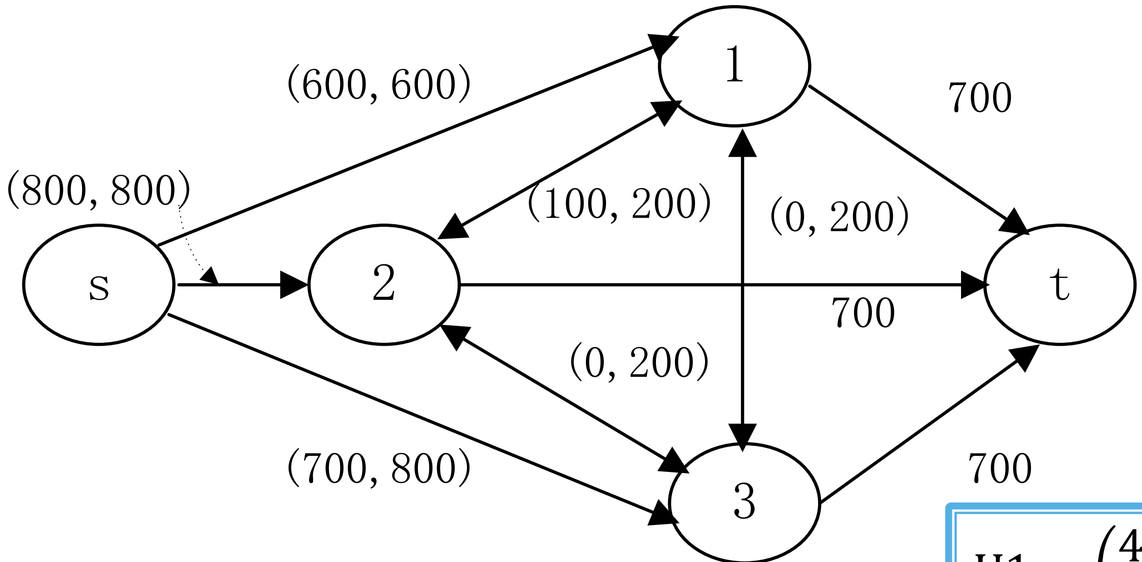
Also v1 has only possible state

$$L11 = \begin{pmatrix} 3 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix} = \emptyset$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \ldots\ldots & v_k - 1 \ldots\ldots & M_n \\ v_1 & \ldots\ldots & m_k \;\ldots\ldots & m_n \end{pmatrix}$$

Find v2 and corresponding L12



$$U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$f2 - e2 = 800 - 100 = 700$$

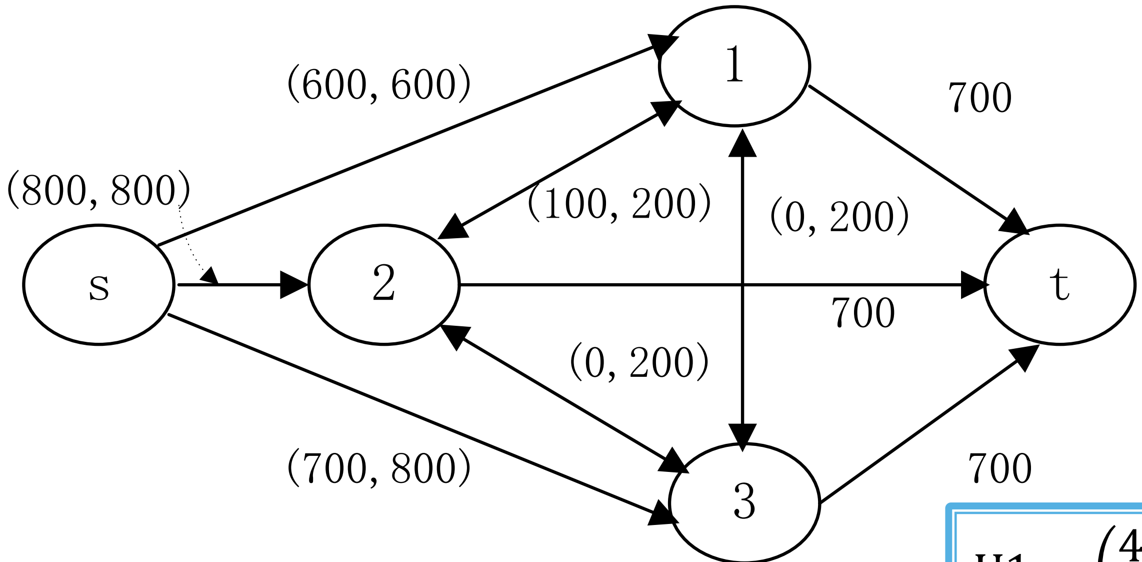
$$v2 = 5\ (800\ capacity)$$

$$L12 = \begin{pmatrix} 4 & 4 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 \dots\dots & M_n \\ v_1 & \dots\dots & m_k \;\; \dots\dots & m_n \end{pmatrix}$$

Find v3 and corresponding L13



$$U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

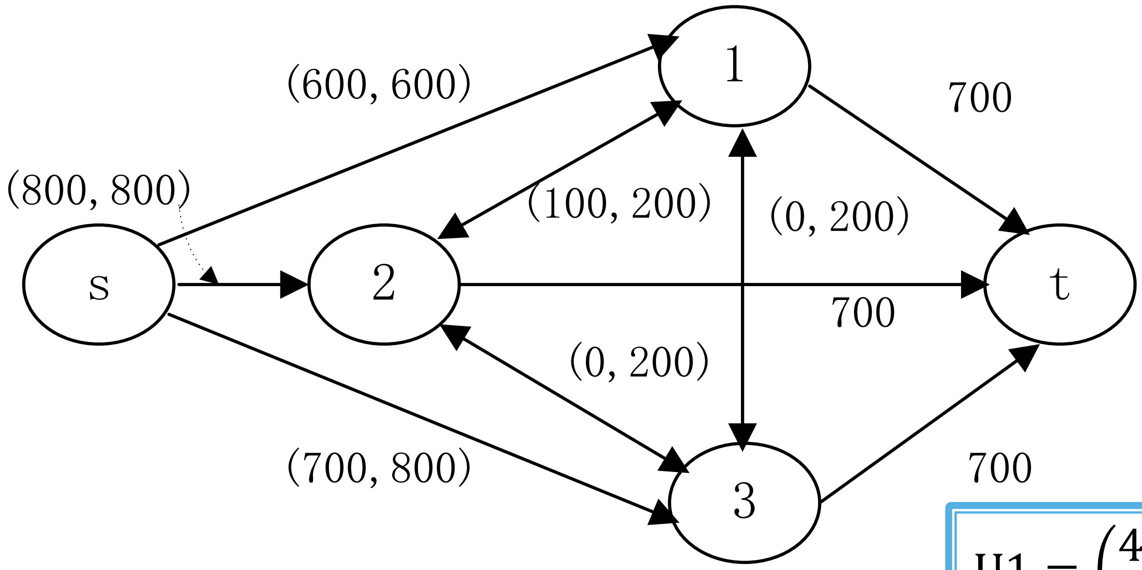
$$f3 - e3 = 700 - 0 = 700$$
$$v3 = 5\ (800\ capacity)$$

$$L13 = \begin{pmatrix} 4 & 5 & 4 & 1 & 1 & 1 \\ 4 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

Example Problem

$$L_k = \begin{pmatrix} M_1 & \dots\dots & v_k - 1 \dots\dots & M_n \\ v_1 & \dots\dots & m_k \quad \dots\dots & m_n \end{pmatrix}$$

Find v4 and corresponding L14



$$U1 = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$f4 - e4 = 100 - 100 = 0$$
$$v4 = 1$$

V4 has only one possible state

$$\text{L14} = \begin{pmatrix} 4 & 5 & 5 & 0 & 1 & 1 \\ 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix} = \emptyset$$

Example Problem

$$U_k = \begin{pmatrix} M_1 & \dots\dots & u_k - 1 \dots\dots & M_n \\ u_1 & \dots\dots & v_k \quad \dots\dots & v_n \end{pmatrix}$$

Find remaining U sets

$$\text{U1} = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix} = 4\ states$$

<u>Trick:</u>

<u>For U sets with few states, you can find out when to stop decomposition by inspection.</u>

$$\text{A} - \text{U1} = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix} = 1\ state$$

$$\text{L12} = \begin{pmatrix} 4 & 4 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix} = 2\ states$$

$$\text{L13} = \begin{pmatrix} 4 & 5 & 4 & 1 & 1 & 1 \\ 4 & 5 & 4 & 1 & 1 & 1 \end{pmatrix} = 1\ state$$

Thus

U sets of U1 are empty

Example Problem

i	C_i (Capacity out)	P_i (Cum. Prob.)	Arc State
1	0 MW	1	5
2	200 MW	0.4780	4
3	400 MW	0.1095	3
4	600 MW	0.0120	2
5	800MW	0.0005	1

Check probability, also prepare for question 3

$$\mathrm{U1} = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(U1) = (P_2 - P_3) * (P_1 - P_3) * (P_1 - P_3) * 1 = 0.2922$$

After decomposition

$$\mathrm{A - U1} = \begin{pmatrix} 4 & 5 & 5 & 1 & 1 & 1 \\ 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(A - U1) = (P_2 - P_3) * (P_1 - P_2) * (P_1 - P_2) * 1 = 0.1004$$

$$\mathrm{L12} = \begin{pmatrix} 4 & 4 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L12) = (P_2 - P_3) * (P_2 - P_3) * (P_1 - P_3) * 1 = 0.1209$$

$$\mathrm{L13} = \begin{pmatrix} 4 & 5 & 4 & 1 & 1 & 1 \\ 4 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L13) = (P_2 - P_3) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.0709$$

$$\sum Prob = 0.2922 = Prob(U1)$$

Example Problem

i	C_i (Capacity out)	P_i (Cum. Prob.)	Arc State
1	0 MW	1	5
2	200 MW	0.4780	4
3	400 MW	0.1095	3
4	600 MW	0.0120	2
5	800MW	0.0005	1

Decompose U2

$$\mathrm{U2} = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 5 & 4 & 4 & 1 & 1 & 1 \end{pmatrix} = 2\ states$$

$$Prob(U2) = (P_1 - P_2) * (P_2 - P_3) * (P_1 - P_3) * 1 = 0.1713$$

Repeat previous procedures

$$\mathrm{A - U2} = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 5 & 4 & 5 & 1 & 1 & 1 \end{pmatrix} = 1\ state$$

$$Prob(A - U2) = (P_1 - P_2) * (P_2 - P_3) * (P_1 - P_2) * 1 = 0.1004$$

$$\mathrm{L23} = \begin{pmatrix} 5 & 4 & 4 & 1 & 1 & 1 \\ 5 & 4 & 4 & 1 & 1 & 1 \end{pmatrix} = 1\ state$$

$$Prob(L23) = (P_1 - P_2) * (P_2 - P_3) * (P_2 - P_3) * 1 = 0.0709$$

$$\sum Prob = 0.1713 = Prob(U2)$$

Example Probler

i	C_i (Capacity out)	P_i (Cum. Prob.)	Arc State
1	0 MW	1	5
2	200 MW	0.4780	4
3	400 MW	0.1095	3
4	600 MW	0.0120	2
5	800MW	0.0005	1

Decompose U3

$$\mathrm{U3} = \begin{pmatrix} 5 & 5 & 4 & 1 & 1 & 1 \\ 5 & 5 & 4 & 1 & 1 & 1 \end{pmatrix} = 1\ state$$

$$Prob(U3) = (P_1 - P_2) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.1004$$

Repeat previous procedures

$$\mathrm{A-U3} = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 5 & 5 & 5 & 1 & 1 & 1 \end{pmatrix} = 1\ state$$

$$Prob(A - U3) = (P_1 - P_2) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.1004$$

$$\sum Prob = 0.1004 = Prob(U3)$$

Example Problem

After level 2 decomposition,
U sets become empty sets, thus state space is fully decomposed.

To answer question 3, summarize previous probabilities

After level 2 decomposition

$$Prob(A) = Prob(A) + Prob(A - U1) + Prob(A - U2) + Prob(A - U3)$$
$$= 0.4435$$

$$Prob(L) = Prob(L1) + Prob(L2) + Prob(L3) + Prob(L12) + Prob(L13)$$
$$+ Prob(L23) = 0.5565$$

$$Prob(U) = 0$$

Example Problem

<u>Question 3:</u> Find the probabilities of A, L and U sets from these two levels of decomposition and tabulate the trend as a function of the level of decomposition

Put into a tabular form

	Level of Decomposition	
	1	2
Prob(A)	0.1422	0.4436
Prob(L)	0.2939	0.5565
Prob(U)	0.5639	0

Example Problem

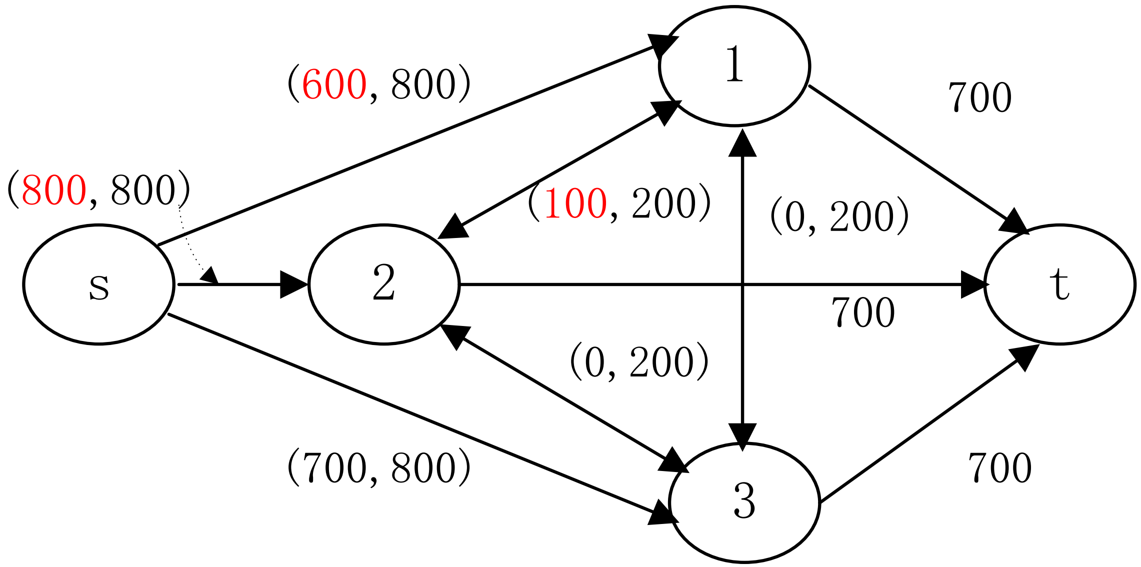
What if we had different u vector at the beginning?

Different results after 1st level of decomposition

But same final results when state space is fully decomposed.

Example Problem

Firstly, find the maximum flow and corresponding u vector



$$u = (4 \quad 5 \quad 5 \quad 1 \quad 1 \quad 1)$$

700 flow needs 800 capacity, thus u1= u2= u3= state 5

Thus, $A = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$

$$A = \begin{pmatrix} M_1 & M_2 & \dots\dots & M_n \\ u_1 & u_2 & \dots\dots & u_n \end{pmatrix}$$

End of 1st level of decomposition

$$A = \begin{pmatrix} 5 & 5 & 5 & 1 & 1 & 1 \\ 4 & 5 & 5 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(A) = (P_1 - P_3) * (P_1 - P_2) * (P_1 - P_2) * 1 = 0.2426$$

$$L1 = \begin{pmatrix} 3 & 5 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, L2 = \begin{pmatrix} 5 & 3 & 5 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, L3 = \begin{pmatrix} 5 & 5 & 3 & 1 & 1 & 1 \\ 4 & 4 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L1) = (P_3) * (P_1) * (P_1) * 1 = 0.1095$$

$$Prob(L2) = (P_1 - P_3) * (P_3) * (P_1) * 1 = 0.0975$$

$$Prob(L3) = (P_1 - P_3) * (P_1 - P_3) * (P_3) * 1 = 0.0868$$

$$Prob(L) = Prob(L1) + Prob(L2) + Prob(L3) = 0.2939$$

$$U2 = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}, U3 = \begin{pmatrix} 5 & 5 & 4 & 1 & 1 & 1 \\ 4 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(U2) = (P_1 - P_3) * (P_2 - P_3) * (P_1 - P_3) * 1 = 0.2922$$

$$Prob(U3) = (P_1 - P_3) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.1713$$

$$Prob(U) = Prob(U1) + Prob(U2) + Prob(U3) = 0.4635$$

$$\sum Prob = 1 = Prob(S)$$

Example Problem

2^{nd} level of decomposition

Decompose U2

$$\mathrm{U2} = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(U2) = (P_1 - P_3) * (P_2 - P_3) * (P_1 - P_3) * 1 = 0.2922$$

After decomposition

$$\mathrm{A - U2} = \begin{pmatrix} 5 & 4 & 5 & 1 & 1 & 1 \\ 5 & 4 & 5 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(A - U2) = (P_1 - P_2) * (P_2 - P_3) * (P_1 - P_2) * 1 = 0.1004$$

$$\mathrm{L21} = \begin{pmatrix} 4 & 4 & 5 & 1 & 1 & 1 \\ 4 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L21) = (P_2 - P_3) * (P_2 - P_3) * (P_1 - P_3) * 1 = 0.1209$$

$$\mathrm{L23} = \begin{pmatrix} 5 & 4 & 4 & 1 & 1 & 1 \\ 5 & 4 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L23) = (P_1 - P_2) * (P_2 - P_3) * (P_2 - P_3) * 1 = 0.0709$$

$$\sum Prob = 0.2922 = Prob(U1)$$

Example Problem

2nd level of decomposition

Decompose U3

$$\mathrm{U3} = \begin{pmatrix} 5 & 5 & 4 & 1 & 1 & 1 \\ 4 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(U3) = (P_1 - P_3) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.1713$$

After decomposition

$$\mathrm{A - U3} = \begin{pmatrix} 5 & 5 & 4 & 1 & 1 & 1 \\ 5 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(A - U3) = (P_1 - P_2) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.1004$$

$$\mathrm{L31} = \begin{pmatrix} 4 & 5 & 4 & 1 & 1 & 1 \\ 4 & 5 & 4 & 1 & 1 & 1 \end{pmatrix}$$

$$Prob(L31) = (P_2 - P_3) * (P_1 - P_2) * (P_2 - P_3) * 1 = 0.0709$$

$$\sum Prob = 0.1713 = Prob(U3)$$

Example Problem

After level 2 decomposition,
U sets become empty sets, thus state space is fully decomposed.

$$Prob(A) = Prob(A) + Prob(A - U2) + Prob(A - U3) = 0.4435$$

$$Prob(L) = Prob(L1) + Prob(L2) + Prob(L3) + Prob(L21) + Prob(L23) + Prob(L31) = 0.5565$$

$$Prob(U) = 0$$

	Level of Decomposition	
	1	2
Prob(A)	0.2426	0.4436
Prob(L)	0.2939	0.5565
Prob(U)	0.4635	0

Example Problem

Different u vector affect result of current level decomposition

But all of them will reach the same result when state space is fully decomposed

	Level of Decomposition	
	1	2
Prob(A)	0.1422	0.4436
Prob(L)	0.2939	0.5565
Prob(U)	0.5639	0

	Level of Decomposition	
	1	2
Prob(A)	0.2426	0.4436
Prob(L)	0.2939	0.5565
Prob(U)	0.4635	0